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LIMITED TRANSLATION SHRINKAGE
ESTIMATION OF LOSS RATES IN
MARINE CORPS MANPOWER MODELS

by

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Limited Translation Shrinkage Estimation of Loss Rates in Marine Corps Manpower Models

by

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ABSTRACT

The use of shrinkage type estimators for the attrition rates in manpower flow models is extended to include the limited translation James-Stein technique introduced by Efron and Morris. The performance of these estimators is compared with that of several "natural" estimators on two scales: the original scale of rates, and the Freeman-Tukey transformation scale which was chosen in order to give shrinkage estimators more efficacy.

Generally, the aggregate methods currently in use are outperformed by MLE, transformed scale cell average, James-Stein, and limited translation James-Stein. The results among the four were mixed when both global and small inventory cell figures of merit were compared. It is felt that better data aggregation will permit limited translation to excell in low inventory estimation.

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I. INTRODUCTION

A. PURPOSE

This is a continuation of a pilot study started by Major D.D. Tucker in a thesis [Ref. 1] submitted at the Naval Postgraduate School in September 1985. The reader is referred to Tucker [Ref. 1] for most of the background information, including a detailed discussion of the Marine Corps officer attrition and promotion structure, the officer manpower planning process, and the attrition rate models explored by Major Tucker. Only information that is immediately pertinent will be repeated.

B. BACKGROUND

The United States Marine Corps has about 20,000 officers. These can be cross classified into 40 military occupational fields (OF), 31 lengths of service (LOS), and 10 grades, or 12,400 categories for manpower planning purposes. About half (6149) of these categories, called hereafter cells, are unoccupied for structural reasons, e.g., due to policy decisions concerning minimum and maximum lengths of service for each grade, and the allowable grades in each OF. Estimates of the attrition rates from these cells support a number of Marine Corps models, and accurate prediction of the rates is basic to effective manpower utilization.

The goal of this pilot study is to find efficient ways to estimate attrition rates (i.e.,the rate of leaving the service, not of changing OF, LOS, or grade) for the officer OF/LOS/grade categories. This is a difficult problem because of the large number of cells with low inventory figures. We will refer to this as the "small cell" problem; it is this small cell problem that is of greatest concern to the builders and users of these manpower models.

Because of the very large number of cells and their heterogeneous nature, it is wise to collect cells into major groups, or aggregates, which conform to certain assumptions concerning their statistical behavior. If these assumptions are at least approximately valid for the aggregates, then certain theoretical models can be used to predict attrition rates more accurately than current practices allow. Amin Elseramegy [Ref. 2] explored the aggregation problem using the CART routine with encouraging results. But these results were not available or usable in a timely fashion to be included in the present study. This thesis and Tucker [Ref. 1] assume valid aggregations can be found, and explore the performance of likely estimation schemes. However, the aggregates used in these pilot studies conform to current Marine Corps practice. These were selected on grounds that conform to organizational and operational considerations, and are unlikely to be related to a choice made on the basis of the statistical modeling behavior.

Current Marine Corps practice places all OF's in four categories: aviation (OF 72, 75), combat support (OF 13, 25, 35), ground combat (OF 03, 08, 18), and other. Aggregates are formed from these categories by taking data by grade. Past attrition rates, from 1977 to the present, weighted by subjective judgement, are used to predict future attrition. An average attrition rate (the grand mean) is computed for the entire aggregate, and this rate is used as an estimate for all cell attrition rates in the aggregate. We expect to improve substantially over this method.

It should be noted that the aviation category used by Tucker [Ref. 1] included only OF 75. For continuity, this is continued in the present work.

C. PROGRESS

Tucker [Ref. 1] showed the James-Stein shrinkage estimator [Refs. 3,4] can

(1) greatly improve on current methods,

(2) improve, in a global sense, over maximum likelihood methods, and

(3) provide estimates for those small cells which have had no attrition, i.e., those cells whose MLE must equal zero.

Present work continues this investigation in that

(1) global measures of efficacy (risk) are decomposed so that the effects of the method can be separately examined for small and large cells, and

(2) a class of extensions of the James-Stein estimation, called limited translation shrinkage estimation, is applied to the aggregates studied by Tucker [Ref. 1].

The main purpose is to sharpen the treatment given the small cells.

D. RESULTS

It appears the limited translation technique adds to the efficacy of the James-Stein estimates (see Chapter IV), in that the estimation of rates for small cell has improved. Also, an estimator , designated the transformed scale cell average (TSCA) which is a version corresponding to zero James-Stein shrinkage, has been shown to be an efficient estimation technique, often outperforming all other schemes examined here. The various methods are quite competitive, and at this time there is no clear choice. We believe that better aggregation methods need to be applied prior to attempting to choose among these methods.

E. ORGANIZATION

Chapter II contains the details of methodology and notation necessary to the present work. A brief summary of James-Stein estimation is presented, with emphasis on its implementation in the present work.

Chapter III explains the limited translation extension, together with the theoretical curves that help anticipate the effect of this option.

Chapter IV contains the numerical summaries and tabulation of the figures of merit (FOM) for the various techniques.

Chapter V thoroughly discusses the results, including recommendations, and lists additional areas needing examination.

The appendices document certain details of interest to the reader desiring a greater in-depth knowledge of the methods.

II. RELATED ESTIMATION METHODS

A. GENERAL

This chapter describes the estimation method currently in use by the Marine Corps, and four other methods of estimation pertinent to the present work.

B. BACKGROUND

As explained by Tucker [Ref. 1], the performance of the estimation schemes are compared on two scales, transformed and original. The transformed scale is the range space of the Freeman-Tukey transform (see Appendix B) that helps stabilize the variance of the ordinary empirical rates assuming they are described by the binomial model. On this scale the transformed quantities are treated as normally distributed random variables with common variance. It is in this setting that the James-Stein estimator is derived and can be expected to perform well. Because the rates are low and because many cells are small, we cannot assert with confidence that the rates on the transformed scale are approximately normal with common variance. The ultimate value must be judged in terms of cross-validation, i.e., comparing the estimates with like transformed values of future actuals. Following Tucker [Ref. 1] we have chosen the first four years of data of the seven available to estimate rates, and the last three years for validation.

Although comparisons on the transformed scale are valuable for purposes of understanding the behavior of shrinkage estimators, they do not supplant the need to study behavior on the original scale. Hence, the transformed estimators must be inverted to estimated rates on the original scale, and then validated against original scale actuals. The traditional chi-square goodness-of-fit

statistic was chosen to do this. It is a weighted sum of squares deviations measure.

The requirement to work on a transformed scale introduces some additional complications. To illustrate the point, consider the following choice. Should the empirical rate for each cell of the four years be transformed to the new scale, or should we first sum the leavers and the inventory over the four years in order to produce a more stable cell rate prior to transforming? It turns out that if the latter is chosen then we have no reasonable way to estimate the within-group variance on the transformed scale. Hence the former is chosen. This done, the transformed quantities are averaged over time to produce a single figure for each cell prior to shrinkage. (This is the TSCA mentioned in Chapter I, Section B.)

On the other hand, the latter figure is still useful since it is the maximum likelihood estimator of the cell rate on the original scale. But there is still the question of how to use it, for comparison purposes, on the transformed scale. We have chosen to use the four year average cell inventory in conjunction with this MLE rate and then apply the arcsine transformation. See equation B.1.

To avoid confusion, an estimate will always be referred to by the name given when initially calculated. For example, the maximum likelihood estimate (MLE) is calculated initially on the original scale, and still will be called the MLE when on the transformed scale. Also, the term "maximum likelihood" may be used to refer to maximum likelihood estimation in the setting at hand, Thus the TSCA ia a set of specific maximum likelihood estimators as it refers to the "IID normal with common variance" setting imposed on the transformed scale.

C. AGGREGATION

The current Marine Corps attrition rate estimation methodology described in Chapter I is an aggregation scheme, and as such has several weaknesses. First, a single rate is applied to all cells in the aggregate. This does not take into account actual differences in cell inventory or losses. The pattern of losses can differ greatly among cells in an aggregate whose composition is arbitrarily determined.

Also, an aggregation scheme has difficulty handling cells that have had zero inventory for the estimation period, i.e., cells whose MLE would be zero. The application of the aggregate rate in this case is clearly an overestimate.

Two constant rates are used in the present work for comparison purposes. The first is the aggregate rate calculated on the original scale, called hereafter the original scale aggregate. This rate is the total losses divided by the total inventory, to be applied to all cells in the aggregate, and is a single number. For comparison on the transformed scale, this rate is mapped into the transformed space using the arcsine transformation

$$x_i = (N_i+0.5)^{\frac{1}{2}}\sin^{-1}(2p-1), \quad i=1,\ldots,K \qquad (2.1)$$

where p is the aforementioned single rate, and N_i are the average cell inventories over time. This results in different transformed cell means (transformed rates) because of differing cell inventories. Generally the subscript i indexes a combination of LOS and OF.

The second rate is calculated on the transformed scale, and is called hereafter the transform aggregate. It is computed by averaging the cell figures that result from applying the the Freeman-Tukey transformation. On the transformed scale this is a single number and in fact is the

grand mean of the TSCA figures. When inverted onto the original scale for comparison purposes using equation A.12 of Appendix A, the average cell inventory is used, resulting in different rates for each cell.

D. MAXIMUM LIKELIHOOD ESTIMATION (MLE)

The present work calculates an MLE for comparison purposes. As stated in Section B above, this rate is the total leavers (over time) divided by the total inventory (over time), using the four year estimation period. This empirical rate is called MLE because it would be the maximum likelihood estimate in the setting of independent Bernoulli trials. We retain this terminology on the transformed scale.

It is well known that this MLE is the best unbiased estimator if the Bernoulli setting is tenable. The problems using the MLE here are threefold. First, the smaller the number of cell trials the greater the variability in the estimation. Thus while the estimate is unbiased, the range of values the estimate can easily assume is large. A stable estimate cannot be made.

Second, this MLE assumes each data set is drawn from identical populations. Service retention is greatly affected by changing economic, political, and social forces. These forces are not constant, and over a period of three to six years the behavior of a cell can change radically. This introduces the yearly update problem. The requirement to drop old data as new data becomes available keeps the cell trials low, and the variability of the estimate high. In short, we are not yet equipped to build an estimation scheme based upon manpower flow model structural conditions.

Third, manpower planning cannot focus on an individual OF/LOS/grade category. Since Marine Corps officers approximate a hierarchical system, the requirement to promote an officer to fill a projected loss of, for example, an infantry Lieutenant Colonel will "ripple" down to a requirement to recruit an infantry Second Lieutenant.

E. TRANSFORMED SCALE CELL AVERAGE (TSCA)

Tucker [Ref. 1] and the present work calculate a TSCA rate. This rate can be viewed as a maximum likelihood estimator calculated on the transformed scale, using transformed inventory and loss data. If the "normal with common variance" model were firmly defensible on the transformed scale, the TSCA would provide the best linear unbiased estimators of the individual cell means. The method is accorded separate treatment because of the excellent results, especially in the near term (one year) validations.

Stein [Ref. 3] in 1955 examined the performance of this maximum likelihood estimator in predicting cell values. He established that if the number of cells is at least three, then maximum likelihood estimation can be improved in an overall sense. The criterion he used was the global loss,

$$L(\theta,a) = \sum_i \ell(\theta_i,a_i), \qquad i = 1,\ldots,k \tag{2.2}$$

where θ is the array of unobservable true cell values and a is the array of predicted cell values. This global loss is the sum across all cells if the individual loss is

$$\ell(\theta_i,a_i) = (\theta_i-a_i)^2. \tag{2.3}$$

where θ_i and a_i are the appropriate values for the i^{th} cell in the aggregate.

F. JAMES-STEIN ESTIMATION

James and Stein [Ref. 4] developed an estimator, called the James-Stein estimator, which reduces the expected value of the global loss, when compared to the cell means. The expected value is called the risk R:

$$R = E[L(\theta,a)] = \sum_i E[(\theta_i-a_i)^2], \qquad \forall\ i. \tag{2.4}$$

Two assumptions were made:

(1) the cell values are normally distributed, and

(2) the within cells variance is constant.

The basic idea of James-Stein estimation is the farther the cell mean is from the overall mean, the greater is the size of the residual error. Note that the cell mean is the TSCA, and the overall mean is the transform aggregate , or grand mean. All means are moved, or shrunk, toward the grand mean. The amount of shrinkage is proportional to the absolute distance from the grand mean, i.e., the greater the absolute distance, the greater is the shrinkage.

There are, however, problems with this method. First, a natural objection is that some cell attrition rates may be far from the grand mean simply because the long term attrition from these cells differ greatly from the majority of cells in the same aggregate. To shrink the attrition rates of these cells toward the grand mean may be erroneous.

In dealing with a sample in the original scale, problems occur when there is no cell loss for the entire estimation period, i.e., when the MLE is zero. Tucker [Ref. 1] handled this by omitting such cells from all comparisons. In the present work these zero loss cells are retained in an effort to view the effect of the various schemes on the small cell rate estimations.

Third, Appendix A demonstrates the Freeman-Tukey transformation does not normalize the cell means or stabilize the variance when the inventory or loss rates are low. Since normality with common variance is the basic assumption of the James-Stein scheme, the reliability of the results must be questioned.

See Appendix A for the James-Stein estimation algorithm as used by Tucker [Ref. 1].

See James and Stein [Ref. 4] and Tucker [Ref. 1] for details on James-Stein estimation.

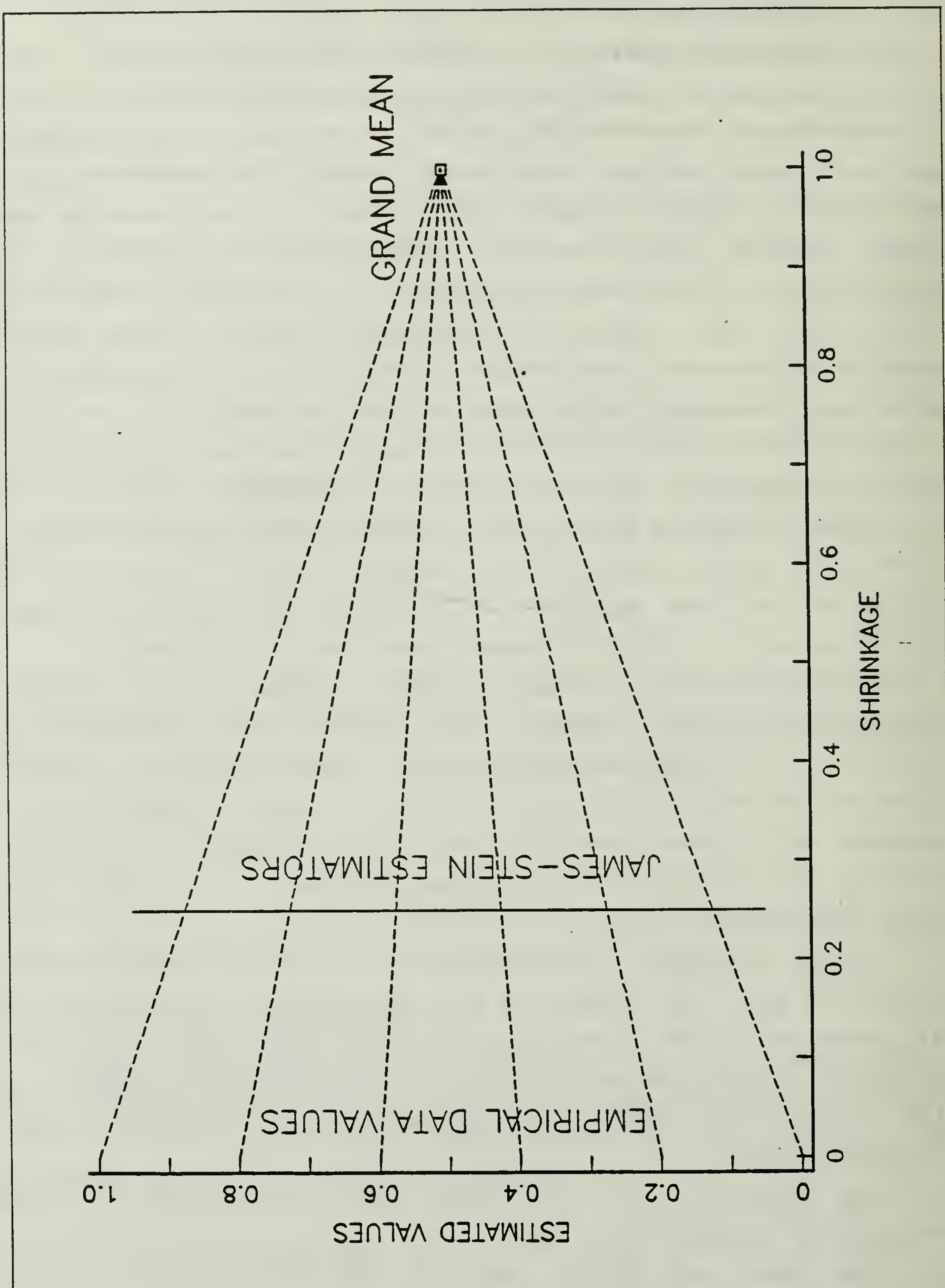

Figure 2.1 James-Stein Shrinkage

G. ROBUST PARAMETRIC EMPIRICAL BAYES ESTIMATION

An alternative method of analysis, unrelated to the schemes investigated here, is the robust parametric empirical Bayes (RPEB) model, suggested by D. P. Gaver. While relatively new, the method has shown promise in the settings to which it has been applied.

Because of time constraints, this model was not implemented. The procedure is a significant departure from the present work, but it may offer important benefits to small cell estimation. Appendix F is a brief description of the technique.

III. LIMITED TRANSLATION JAMES-STEIN ESTIMATION

A. GENERAL

This chapter discusses the limited translation model, and the validation approaches taken in this paper.

As stated in Chapter I, it is intuitively unsettling to shrink all empirical cell rates toward the grand mean by the same affine translation. Also, one must question whether the risk could be further reduced from that of the James-Stein estimator. Two articles published by Efron and Morris [Refs. 5,6] present such a method: limited translation of the James-Stein estimator.

To compromise between James-Stein and TSCA estimation, and to limit the translation of extreme values, an interval [-C,C] centered about the grand mean is established. Inside this interval all rates are translated using full James-Stein shrinkage. Outside this interval the amount of shrinkage is reduced the farther cell values get from the interval. The shrinkage approaches zero in the limit.

To get an intuitive feel for the differences between James-Stein and limited translation James-Stein estimation, compare Figures 2.1 and 3.1. Figure 2.1 shows how the James-Stein technique shrinks all values toward the grand mean. Figure 3.1 shows how limited translation estimation reduces the shrinkage outside a certain range of values centered about the grand mean.

Theoretically, limited translation estimation, by shrinking some cells, will slightly increase the global risk over that of the James-Stein estimator. This increase is acceptable since the individual cell risk of extreme inventory cells is decreased. This means the estimators of the small cell attrition rates improve, usually significantly, at a small cost to the middle inventory cells.

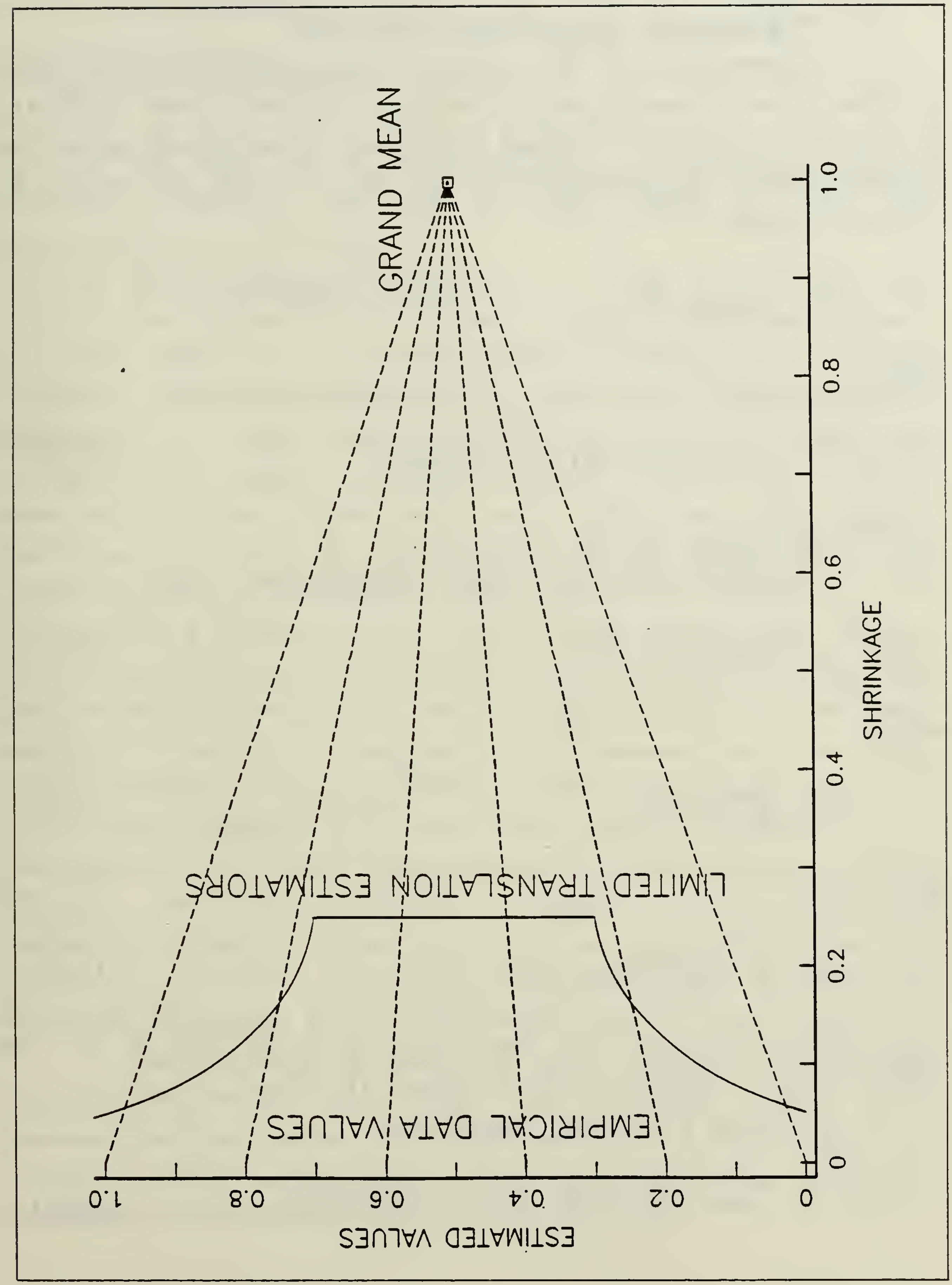

Figure 3.1   Limited Translation James-Stein Shrinkage

B. THE SHRINKAGE FACTOR

1. The Limited Translation Algorithm

Conversion of the James-Stein estimation algorithm to limit the translation is straightforward. From Appendix A, after the attrition rates have been transformed using the Freeman-Tukey transformation, the James-Stein estimator P_J is calculated,

$$P_J = \bar{\bar{X}} + C_J(\bar{X}_{i\bullet} - \bar{\bar{X}}), \tag{3.1}$$

where

$$C_J = 1 - (K-3)SSE/[(K(T-1)+2)SSB] \tag{3.2}$$

is the shrinkage factor.

To modify C_J for limited translation, let

$$\rho(u) = \text{minimum } (1, d/u^{\frac{1}{2}}) \tag{3.3}$$

where

$$u = (\bar{X}_{i\bullet} - \bar{\bar{X}})^2/(A+1). \tag{3.4}$$

Note that

$$A = (K(T-1)+2)SSB/(K-3)SSE - 1 \tag{3.5}$$

and is the variance of the prior distribution of θ. The value of d is chosen from the interval $[0,\infty]$.

The new shrinkage factor is

$$C_{LJ} = 1 - \rho(u)[(K-3)SSE/((K(T-1)+2)SSB)] \tag{3.6}$$

and the new estimator is

$$P_{LJ} = \bar{\bar{X}} + C_{LJ}(\bar{X}_{i\bullet}-\bar{\bar{X}}). \tag{3.7}$$

If $d/u^{\frac{1}{2}}$ is equal to or greater than 1, then full shrinkage occurs. However, if $d/u^{\frac{1}{2}}$ is less than 1, then full shrinkage does not occur.

2. The Shrinkage Interval

The choice of a d value is important. The larger d is, the larger the interval [-C,C] becomes. If d becomes infinite, then the estimator P_{LJ} is simply the James-Stein estimator P_J. This would result in no change in the global or individual risks. However, if $d/u^{\frac{1}{2}}$ is less than one for some cell values, then those cells will not fully shrink, and there will be an improvement in the small cell individual risk. If d equals zero, then the interval [-C,C] shrinks to a point at the value of the grand mean, and all cell values shrink to the grand mean. In this case the variability of the cell rates is zero Thus, we want some intermediate value of d. Appendix E discusses the theoretical implications of d, and methods of choosing values.

The effects of d can be directly observed in the pattern of the limited translation shrinkage factors. Within [-C,C], the cell shrinkage factors will equal the James-Stein factor. The farther one gets from the full shrinkage interval, the smaller the shrinkage factors are. Note the shrinkage factor approaches zero in the limit.

Caution must be exercised when viewing the shrinkage factor pattern in this light. The interval [-C,C] is an interval of cell values, in this case cell means. Therefore, if the cell value pattern is not apparent, the pattern of shrinkage factors will also not be apparent.

C. VALIDATION

The development of the James-Stein scheme took place in a restrictive setting. It is useful to state that setting in a form most closely associated with our problem:

$$X_{ij} = \mu_i + e_{ij}, \qquad i=1,\ldots,k \text{ and } j=1,\ldots,n \qquad (3.8)$$

with the $\{e_{ij}\}$ as IID $N(0,\sigma^2)$ random variables. Since this is the same setting as that of the one way analysis of variance, it is convenient to use the quantities and notation of ANOVA. Specifically, let

$$SSE = \sum_i \sum_j (X_{ij} - \overline{X}_{i\bullet})^2, \qquad \forall\ i,j, \qquad (3.9)$$

$$SSB = n\sum_i (\overline{X}_{i\bullet} - \overline{X}_{\bullet\bullet})^2, \qquad \forall\ i, \qquad (3.10)$$

$$\overline{X}_{i\bullet} = (1/n)\sum_j X_{ij}, \qquad \forall\ i,j, \text{ and} \qquad (3.11)$$

$$\overline{X}_{\bullet\bullet} = (1/k)\sum_i \overline{X}_{i\bullet}, \qquad \forall\ i. \qquad (3.12)$$

Since we will be using equations (3.2), (7.3), (7.4), and (7.7) of reference 5, we record the identifications

$$\hat{\sigma}^2 = SSE/(2+k(n-1)), \qquad (3.13)$$

$$V = SSB/n, \qquad (3.14)$$

$$\bar{X}_c = \bar{X}_{c\bullet} \text{ for } c=1,\ldots,k, \tag{3.15}$$

$$\bar{\bar{X}} = \bar{X}_{\bullet\bullet}, \text{ and} \tag{3.16}$$

$$\rho(u) = \text{minimum}(1, d/u^{\frac{1}{2}}). \tag{3.17}$$

On the other hand our cell attrition data is fairly modeled with the binomial distribution [Ref. 1] such that

(1) y_{ij} = number of leavers in cell i during period j,

(2) N_{ij} = central inventory,

and the empirical cell attrition rates are

$$p_i = (\sum_j y_{ij})/(\sum_j N_{ij}), \qquad \forall_{ij}. \tag{3.18}$$

Because of the large number of small cells, the above empirical probabilities are unstable and it should be possible to improve the stability by shrinking them.

If we were to treat the p_i's as the $X_{i\bullet}$'s in the ANOVA setting, we encounter some flaws. Namely, the common variance and normality assumptions are severly compromised, and there is no obvious way to estimate σ^2. The same flaws are present if we back off and use the cell empirical rates,

$$y_{ij}/N_{ij}. \tag{3.19}$$

in the role of X_{ij}.

In hopes of giving relief to this problem we (see reference 1) have chosen to use the Freeman-Tukey variance stabilization transform

$$z_{ij} = 0.5(N_{ij}+0.5)^{\frac{1}{2}}\{\sin^{-1}[2(y_{ij})/(n_{ij}+1)-1] + \sin^{-1}[2(y_{ij}+1)/(n_{ij}+1)-1]\} \tag{3.20}$$

in the role of the X_{ij}. For the small cells we still have the flaws but not to as great an extent as before. See Appendix B.

This leads us to the perform some validation computations using the transformed scale, i.e., the sum of squared error risk function will be computed with the error input given by the difference between the z_{it} (for tϵa cross validation year) and the James-Stein shrunk value z_i computed from (7.7) of reference 5, i.e.,

$$\hat{z}_i = \bar{z}_{\bullet\bullet} + C(\bar{z}_{i\bullet}-\bar{z}_{\bullet\bullet}) \tag{3.21}$$

and

$$C = (K-3)SSE/(2+K(n-1))SSB. \tag{3.22}$$

These risks will serve to tell us whether the shrinkage technique is behaving as expected in spite of the rough treatment given the assumptions in the setting.

It is also important to make validation computations in the original scale. Even though the risks in the transformed scale look attractive, the attrition rate estimates must be converted back to the original scale. For this purpose we propose the chi-square statistic

$$\sum_i (y_{it}-N_{it}p_i)^2/N_{ij}p_i(1-p_i), \qquad \forall\ i \tag{3.23}$$

where y_{it} and N_{it} are the leavers and central inventory counts for the i^{th} cell in the t^{th} validation period, and

$$\hat{p}_i = .5[1-\sin(\hat{z}_i/(\bar{N}_i+.5)^{\frac{1}{2}}], \text{ and} \tag{3.24}$$

$$\bar{N}_i = (1/n)\sum_j N_{ij}, \qquad \forall\ i,j \tag{3.25}$$

provided the argument of the sine function belongs to $[-\pi/2, \pi/2]$. Outside this range, we use $p_i = 0$ or 1 according to whether the argument is below $-\pi/2$ or above $\pi/2$, respectively. The above value, $\hat{p}_i$, is what we will call the James-Stein attrition rate generator.

Above we have described two validation risk calculations, one in the transformed scale and one in the original scale. Let us now address the question of "To what are these risks to be compared?". The general answer is to make like calculations for the other estimation schemes: original and transformed scale aggregates, TSCA, maximum likelihood, and the limited translation. To be specific, we must address some details. The first four will be taken up here and the limited translation modification will be discussed later.

We will discuss the easiest cases first: aggregate and maximum likelihood in the original scale. Let i index all cells in the aggregate. We define an indicator variable

$$D_i = \begin{cases} 1 & \text{if } N_i > 0 \\ 0 & \text{if } N_i = 0 \end{cases}, \quad \forall\, i \tag{3.26}$$

then

$$k = \sum_i D_i \;, \quad \forall\, i. \tag{3.27}$$

k is thus the number of cells in the aggregate with non-zero inventory.

Now, if we define another indicator variable

$$D'_i = \begin{cases} 1 & \text{if } N_i > 0 \text{ and } p_{MLE}(i) \neq 0 \text{ or } 1 \\ 0 & \text{if } N_i = 0 \text{ or } p_{MLE}(i) = 0 \text{ or } 1 \end{cases}, \quad \forall\, i \tag{3.28}$$

then

$$k' = \sum_i D'_i \;, \quad \forall\, i. \tag{3.29}$$

k' is thus the number of cells in the aggregate with non-zero inventory and the MLE not equaling zero or one.

The aggregate attrition rate is

$$p = (\sum_i \sum_j y_{ij})/(\sum_i \sum_j N_{ij}), \qquad \forall\ i,j. \tag{3.30}$$

and this value is inserted in place of p_i in the chi-square statistic. For the maximum likelihood method we use instead

$$p_{MLE}(i) = (\sum_j y_{ij})/(\sum_j N_{ij}), \qquad \forall\ i,j. \tag{3.31}$$

In both cases we must remove all terms in the sum for which N_{ij} is zero. This has the effect of reducing k. Also, since we may have some $p_{MLE}(i) = 0$ or 1, these terms are removed, leading to k' risk terms in the sum. It is recommended that such results be multiplied by k/k' in order to provide a fair comparison with methods that provide positive non-unity estimators for all cells.

Turning to the risk calculation in the transformed scale, there are some alternative ways to decide what we call an aggregate or a TSCA estimator. The question arises because to each cell there is associated two numbers, y_{ij} and N_{ij}.

If performance in the transformed scale were the only concern, we would simply use $\bar{z}_{\bullet\bullet}$ for the transformed aggregate and $\bar{z}_{i\bullet}$ for the TSCA estimator. This in fact was done by Tucker [Ref. 1]. But because of the varying inventory, these values, $\bar{z}_{\bullet\bullet}$ and $\bar{z}_{i\bullet}$, do not correspond to p and $p_{MLE}(i)$. For purposes of the present study it was decided to cast these two values into the transformed scale:

$$\begin{aligned} z_a(i) = 0.5(\bar{N}_i+0.5)^{\frac{1}{2}}\{&\sin^{-1}[2(\bar{N}_i\bar{p})/(\bar{N}_i+1)-1] \\ &+ \sin^{-1}[2(\bar{N}_i\bar{p}+1)/(\bar{N}_i+1)-1]\} \end{aligned} \tag{3.32}$$

where $\bar{N}_i$ is the average inventory for the i^{th} cell. This choice leads to the original scale aggregation in the transformed scale varying form cell to cell. It is fair to do this because the varying inventory allowance is a part of the process that helps stabilize the value in the transformed scale.

For the same reason, and writing p_i for $p_{MLE}(i)$, we use

$$z_{MLE}(i) = 0.5(\bar{N}_i+0.5)^{\frac{1}{2}}\{\sin^{-1}[2(\bar{N}_i p_i)/(\bar{N}_i+1)-1] + \sin^{-1}[2(\bar{N}_i p_i+1)/(\bar{N}_i+1)-1]\} \quad (3.33)$$

for the maximum likelihood estimator in the transformed scale.

IV. RESULTS

A. GENERAL

This chapter displays various data results for the estimation methods compared in this paper.

B. SHRINKAGE FACTORS

The shrinkage factors for the James-Stein (JS) and the limited translation James-Stein (LTJS) methods are presented, by aggregate, in Table 1 thru Table 3. An aggregate is a specific OF group and grade, e.g., the aviation First Lieutenants, or the combat support Lieutenant Colonels. One James-Stein factor is calculated for an entire aggregate, while the limited translation technique assigns a factor to each cell based on the inventory and the chosen value of d. If the distributional assumptions are reasonable, a properly selected d will force the middle inventory cells to have equal James-Stein and limited translation factors, with the limited translation factors getting smaller the farther one is from this middle inventory range. As discussed in Appendix B, these assumptions were not met. Use of a graph of relative savings loss versus d, for different validation years, yielded inconclusive results, except for the aviation First Lieutenant aggregate. See Appendix E. Since a point of emphasis of this study is small cell estimation, d values were chosen to force the limited translation shrinkage factors to act in accordance with theoretical patterns, as discussed in Chapter III and reference 5. The purpose was to evaluate the resulting small cell risk values. See Table 4.

The shrinkage factors exhibit interesting behavior. Within the interval [-C,C] the limited translation shrinkage equals James-Stein. Once outside the interval, the limited

translation factor is reduced. For example, consider Table 2. The code 7 (Engineers) Lieutenant Colonels have a James-Stein shrinkage of .1720. All limited translation factors are less than or equal to this. Those that are equal have cell means (on the transformed scale) within [-C,C]. Note that an isolated cell can fall within this interval, such as the factors for LOS's of 15, 17, 20, and 29. For those cell with means outside [-C,C], the respective limited translation shrinkage is less than .1720. See Chapter III, Section B, Subsection 2. This behavior is non-monotone with respect to LOS, and upon reflection, should be anticipated. The pattern of officer losses is tied to contract expirations, reduced promotional expectations, and retirement options.

C. FIGURES OF MERIT

Table 5 thru Table 7 display the figures of merit for the six estimation schemes in transformed and original space. As expected, both aggregate estimation methods have uniformly higher risks than the other techniques.

Several points are worth noting. First, in the transformed scale, limited translation ranks first overall in lowest risk. This is contrary to the theory as developed by Efron and Morris [Refs. 5,6]. As noted in Chapter 3, limited translation should lower the individual cell risks of those cells outside the interval [-C,C], but at the cost of an increase in global risk.

When the FOM is examined by grade, TSCA is always best for First Lieutenants, and James-Stein is always best for Lieutenant Colonels. This effect appears to be unchanging.

In the original scale, rankings change over time. For 1981, TSCA is best overall and for each grade. This is also true for 1982, but to a lesser degree. In 1983, limited translation is the best overall and for each grade.

TABLE 1

AVIATION SHRINKAGE FACTORS

CODE 38

	LOS	1st LT	LTCOL
JS		.0128	.0322
LTJS	0	.0093	.0197
	1	.0093	.0197
	2	.0128	.0197
	3	.0085	.0197
	4	.0072	.0197
	5	.0128	.0197
	6	.0128	.0197
	7	.0128	.0197
	8	.0128	.0197
	9	.0128	.0197
	0	.0128	.0197
	11	.0128	.0197
	12	.0128	.0197
	13	.0104	.0197
	14	.0096	.0197
	15	.0093	.0234
	16	.0093	.0263
	17	.0093	.0322
	18	.0093	.0177
	19	.0093	.0127
	21	.0093	.0256
	22	.0093	.0205
	23	.0093	.0322
	24	.0093	.0322
	25	.0093	.0322
	26	.0093	.0322
	27	.0093	.0274
	28	.0093	.0254
	29	.0093	.0250
	30	.0093	.0192

The excellent performance of the TSCA estimate is worth noting. TSCA may be thought of as the James-Stein estimator with zero shrinkage. Recognizing that shrinkage is an estimated parameter, we are surprised in those cases for which the estimated shrinkage is large, and TSCA outperforms James-Stein and limited translation. See Tables 3 and 6.

D. SMALL CELL FIGURES OF MERIT

The risk associated with the small cells was investigated to determine which technique best predicts small cell

TABLE 2

COMBAT SUPPORT SHRINKAGE FACTORS

		CODE 7		CODE 13		CODE 20	
	LOS	1LT	LTCOL	1LT	LTCOL	1LT	LTCOL
JS		.1947	.1720	.1947	.1720	.1947	.1720
LTJS	0	.0645	.1531	.0645	.1531	.0645	.1531
	1	.0645	.1531	.0645	.1531	.0645	.1531
	2	.0424	.1531	.0501	.1531	.1947	.1531
	3	.0426	.1531	.0459	.1531	.1001	.1531
	4	.0323	.1531	.0421	.1531	.1947	.1531
	5	.0454	.1531	.1383	.1531	.1947	.1531
	6	.1947	.1531	.0978	.1531	.1368	.1531
	7	.1947	.1531	.0956	.1531	.1083	.1531
	8	.0656	.1531	.1947	.1531	.0422	.1531
	9	.1947	.1531	.1116	.1531	.0528	.1531
	10	.1947	.1531	.0404	.1531	.1539	.1531
	11	.1947	.1531	.0551	.1531	.1947	.1531
	12	.1947	.1531	.1044	.1531	.1947	.1531
	13	.1947	.1531	.1947	.1531	.1947	.1531
	14	.1387	.1531	.1743	.1531	.1947	.1531
	15	.1556	.1720	.0783	.1531	.1190	.1531
	16	.1947	.1531	.0645	.1720	.1947	.1720
	17	.1947	.1720	.0645	.1720	.1947	.1720
	18	.1947	.0898	.0645	.0997	.1947	.1720
	19	.1947	.0718	.0833	.0645	.1947	.1720
	20	.1947	.1720	.0833	.1353	.0965	.1720
	21	.1947	.0919	.0645	.1383	.1947	.1720
	22	.1947	.1447	.0645	.1600	.1277	.1720
	23	.1947	.1575	.0645	.1671	.1539	.1720
	24	.1084	.1720	.0645	.1720	.0980	.1531
	25	.0833	.1720	.0645	.1720	.0995	.1531
	26	.0833	.1720	.0645	.1720	.0995	.1531
	27	.0645	.1061	.0645	.1720	.0974	.1531
	28	.0645	.1418	.0645	.1720	.0727	.1531
	29	.0645	.1720	.0645	.1720	.0727	.1720
	30	.0645	.1555	.0645	.1720	.1077	.1253

attrition since the global figures of merit in Tables 4 thru 6 may be disguising what is happening in the small cells. The small cell figures of merit tables record the small cell contribution to the global risk. Table 8 thru Table 13 display these results. Two average inventory ranges were examined: zero to five, and six to ten.

For the range zero to five, in the transformed scale, the results are varied. The TSCA method ranks first for 1981, limited translation first for 1982, and James-Stein

TABLE 3

GROUND COMBAT SHRINKAGE FACTORS

		CODE 3		CODE 5		CODE 10	
	LOS	1LT	LTCOL	1LT	LTCOL	1LT	LTCOL
JS		.0796	.0392	.0796	.0392	.0796	.0392
LTJS	0	.0480	.0241	.0480	.0241	.0480	.0241
	1	.0507	.0241	.0480	.0241	.0480	.0241
	2	.0343	.0241	.0441	.0241	.0796	.0241
	3	.0268	.0241	.0527	.0241	.0796	.0241
	4	.0313	.0241	.0744	.0241	.0796	.0241
	5	.0281	.0241	.0641	.0241	.0796	.0241
	6	.0796	.0241	.0796	.0241	.0796	.0241
	7	.0796	.0241	.0796	.0241	.0732	.0241
	8	.0796	.0241	.0796	.0241	.0796	.0241
	9	.0796	.0241	.0796	.0241	.0796	.0241
	10	.0796	.0241	.0796	.0241	.0796	.0241
	11	.0796	.0241	.0796	.0241	.0796	.0241
	12	.0796	.0241	.0796	.0241	.0753	.0241
	13	.0796	.0241	.0796	.0241	.0631	.0241
	14	.0796	.0241	.0657	.0241	.0507	.0241
	15	.0663	.0241	.0537	.0241	.0480	.0241
	16	.0654	.0392	.0537	.0368	.0480	.0275
	17	.0480	.0217	.0480	.0392	.0480	.0392
	18	.0480	.0092	.0480	.0191	.0480	.0392
	19	.0480	.0082	.0480	.0208	.0480	.0392
	20	.0480	.0147	.0480	.0392	.0480	.0392
	21	.0480	.0118	.0480	.0291	.0480	.0392
	22	.0480	.0165	.0480	.0392	.0480	.0392
	23	.0480	.0215	.0480	.0392	.0480	.0392
	24	.0480	.0392	.0480	.0386	.0480	.0383
	25	.0480	.0392	.0480	.0392	.0480	.0271
	26	.0480	.0392	.0480	.0233	.0480	.0389
	27	.0480	.0392	.0480	.0221	.0480	.0309
	28	.0480	.0392	.0480	.0267	.0480	.0323
	29	.0480	.0392	.0480	.0213	.0480	.0286
	30	.0480	.0273	.0480	.0204	.0480	.0283

first for 1983. In the original scale TSCA is first for 1981, James-Stein first for 1982, and limited translation first for 1983. The most surprising result is that no clear pattern has emerged, other than the consistently poor performance of the two aggregate estimation schemes. The limited translation technique has not resulted in a uniform lowering of the small cell risk. In fact, in about one-half of the cases James-Stein estimation resulted in a lower small cell risk than did limited translation.

TABLE 4		
D VALUES USED IN STUDY AGGREGATES		
	1^{st} LT	LTCOL
Aviation	.8	.4
Combat Support	.2	.4
Ground Combat	.5	.3

Table 14 thru Table 19 list the small cell percentages of total risk. These results are useful in determining the actual contribution of the small cells to the total risk for the specific aggregate, estimation scheme, and year.

E. ATTRITION RATES

Since the ultimate purpose of this study is to produce attrition rates, the rates generated by the six estimation methods are presented in Table 20 thru Table 33. The differing results of each method are apparent. Of interest is the relative agreement among MLE,TSCA, James-Stein, and limited translation, when compared to the two aggregate methods.

Also, we are comforted that the attrition rate patterns follow both the pattern of the raw attrition, and experience. Experience tells us that, for First Lieutenants, no attrition occurs within LOS's of zero or one because there are no First Lieutenants with such LOS's. A peak in attrition should occur around the eight year point, because most officers have been promoted to Captain, and those remaining resign to pursue civilian careers. Another peak should occur at twenty years since at this point First Lieutenants who have enlisted time start to retire.

Tucker [Ref. 1] demonstrated this loss pattern in a series of graphs (Figures A.1 thru A.10 in reference 1) which display raw loss rates for selected grades and OF's. These graphs show an increase in attrition rate peaking at eight years for First lieutenants, and again at twenty years.

The rates for TSCA, James-Stein, and limited translation follow this pattern. MLE does also, in general. However, MLE will predict zero attrition if the cells have been empty for the entire estimation period, as it does in Table 22. The predicted rates for LOS 13 thru 19 are zero, while the above three schemes estimate rates ranging from .07 to .09.

The aggregate methods do not display this pattern. In fact, the translated scale aggregate estimate is at a local minimum when the above schemes reach local maximums. The original scale aggregate , of course, remain constant except for those cells forced to zero by zero cell inventories.

TABLE 5

AVIATION FIGURES OF MERIT

	1981	1982	1983
	TRANSFORMED FOM		
1st LT			
AGG ORIG	6.405	9.645	11.393
AGG TRANS	197.621	202.022	208.808
MLE	3.914	9.981	10.420
TSCA	3.461	9.574	10.042
JS	3.678	9.768	10.318
LTJS	3.642	9.764	10.279
LTCOL			
AGG ORIG	9.957	17.997	15.488
AGG TRANS	25.093	29.506	29.310
MLE	4.366	9.058	8.210
TSCA	5.777	10.967	10.394
JS	5.737	10.911	10.355
LTJS	5.744	10.901	10.340
	ORIGINAL FOM		
1st LT			
AGG ORIG	31.499	33.146	57.504
AGG TRANS	310.625	286.068	566.903
MLE	24.455	48.375	57.475
TSCA	19.333	34.678	49.105
JS	22.094	39.596	51.040
LTJS	21.081	40.590	50.690
LTCOL			
AGG ORIG	74.697	110.793	50.456
AGG TRANS	19276.410	248.171	4894.164
MLE	34.473	56.304	22.116
TSCA	37.974	51.410	29.613
JS	43.187	54.901	33.736
LTJS	43.802	57.964	35.619

MAXIMUM LIKELIHOOD COMPARISON FACTOR

	k	k'	k/k'
1st LT	13	11	1.1818
LTCOL	16	13	1.2308

TABLE 6

COMBAT SUPPORT FIGURES OF MERIT

	1981	1982	1983
	TRANSFORMED FOM		
1st LT			
AGG ORIG	1.528	1.842	1.385
AGG TRANS	3.308	3.722	3.238
MLE	2.273	2.843	2.409
TSCA	1.383	2.008	1.468
JS	1.470	2.066	1.532
LTJS	1.428	2.045	1.482
LTCOL			
AGG ORIG	1.316	1.701	1.755
AGG TRANS	2.601	2.898	2.981
MLE	1.589	2.157	2.443
TSCA	.910	1.689	1.752
JS	.809	1.514	1.556
LTJS	.831	1.560	1.611
	ORIGINAL FOM		
1st LT			
AGG ORIG	79.521	74.294	90.992
AGG TRANS	157.641	131.044	27231.380
MLE	123.508	128.483	104.668
TSCA	73.190	80.021	122.850
JS	77.802	81.133	91.139
LTJS	80.324	83.379	68.079
LTCOL			
AGG ORIG	34.631	48.780	104.748
AGG TRANS	48.775	44.739	180.760
MLE	57.249	66.996	48.468
TSCA	27.773	40.413	87.428
JS	27.804	36.758	54.514
LTJS	35.847	37.122	42.044

MAXIMUM LIKELIHOOD COMPARISON FACTOR

	k	k'	k/k'
1st LT	70	33	2.1212
LTCOL	40	23	1.7391

TABLE 7

GROUND COMBAT FIGURES OF MERIT

	1981	1982	1983
	TRANSFORMED FOM		
1st LT			
AGG ORIG	2.774	3.609	5.720
AGG TRANS	18.280	19.064	22.044
MLE	2.693	4.258	6.178
TSCA	1.957	3.447	5.231
JS	2.169	3.629	5.505
LTJS	2.045	3.482	5.334
LTCOL			
AGG ORIG	3.692	3.596	3.783
AGG TRANS	13.108	12.209	12.546
MLE	1.252	2.029	2.925
TSCA	1.598	2.414	3.333
JS	1.584	2.332	3.223
LTJS	1.567	2.367	3.264
	ORIGINAL FOM		
1st LT			
AGG ORIG	79.406	80.340	99.979
AGG TRANS	288.193	255.215	321.524
MLE	111.881	145.280	170.424
TSCA	69.166	92.226	106.611
JS	83.893	98.672	117.320
LTJS	84.775	101.875	121.432
LTCOL			
AGG ORIG	100.321	118.816	111.066
AGG TRANS	591.887	342.375	30663.610
MLE	42.304	43.437	56.710
TSCA	37.521	49.664	57.169
JS	42.460	50.712	61.459
LTJS	45.192	50.940	55.562

MAXIMUM LIKELIHOOD COMPARISON FACTOR

	k	k'	k/k'
1st LT	45	35	1.2857
LTCOL	46	35	1.3143

TABLE 8

AVIATION SMALL CELL FOM
(INV ≤ 5)

	1981	1982	1983
		TRANSFORMED FOM	
1st LT			
AGG ORIG	0.069	0.021	0.017
AGG TRANS	6.612	4.682	5.956
MLE	0.176	0.001	0.083
TSCA	0.021	0.069	0.0001
JS	0.031	0.054	0.002
LTJS	0.029	0.056	0.001
LTCOL			
AGG ORIG	0.434	1.015	0.493
AGG TRANS	2.612	3.628	2.507
MLE	0.277	0.640	0.136
TSCA	0.259	0.617	0.328
JS	0.265	0.637	0.308
LTJS	0.266	0.630	0.314
		ORIGINAL FOM	
1st LT			
AGG ORIG	5.277	0.146	0.504
AGG TRANS	0	0	0
MLE	0	0	0
TSCA	0.882	0.520	0.520
JS	1.432	0.386	0.386
LTJS	0.436	0.136	0.136
LTCOL			
AGG ORIG	6.034	12.751	13.535
AGG TRANS	98.746	138.781	4814.621
MLE	5.846	8.400	1.587
TSCA	3.355	6.112	5.264
JS	5.960	8.279	4.637
LTJS	4.870	7.719	2.737

TABLE 9

AVIATION SMALL CELL FOM
($6 \le INV \le 10$)

	1981	1982	1983
		TRANSFORMED FOM	
1st LT			
AGG ORIG	0.139	0.136	0.0001
AGG TRANS	4.960	4.960	3.500
MLE	0.085	0.085	0.004
TSCA	0.072	0.072	0.008
JS	0.086	0.086	0.004
LTJS	0.086	0.086	0.004
LTCOL			
AGG ORIG	1.966	0.588	0.146
AGG TRANS	2.777	0.711	0.195
MLE	1.096	0.134	0.00001
TSCA	1.535	0.448	0.001
JS	1.541	0.425	0.0004
LTJS	1.541	0.432	0.0004
		ORIGINAL FOM	
1st LT			
AGG ORIG	1.564	1.564	0.331
AGG TRANS	0	0	0
MLE	0.692	0.692	0.556
TSCA	0.459	0.459	0.575
JS	0.643	0.643	0.507
LTJS	0.594	0.594	0.468
LTCOL			
AGG ORIG	36.632	9.338	0.332
AGG TRANS	19108.090	40.132	0.726
MLE	9.495	0.821	0.462
TSCA	15.302	3.704	0.713
JS	12.999	2.586	0.680
LTJS	9.943	1.992	0.453

TABLE 10

COMBAT SUPPORT SMALL CELL FOM
(INV ≤ 5)

	1981	1982	1983
	TRANSFORMED FOM		
1st LT			
AGG ORIG	0.509	0.305	0.473
AGG TRANS	0.673	0.355	0.518
MLE	0.945	0.929	1.229
TSCA	0.436	0.369	0.632
JS	0.432	0.326	0.564
LTJS	0.445	0.360	0.606
LTCOL			
AGG ORIG	0.791	1.073	1.055
AGG TRANS	0.614	0.612	0.897
MLE	1.234	1.736	2.164
TSCA	0.753	1.391	1.479
JS	0.650	1.159	1.241
LTJS	0.670	1.217	1.308
	ORIGINAL FOM		
1st LT			
AGG ORIG	35.470	26.300	51.054
AGG TRANS	54.725	19.755	27137.010
MLE	27.405	13.489	24.454
TSCA	30.167	24.187	82.679
JS	30.208	19.083	46.552
LTJS	36.180	26.113	29.901
LTCOL			
AGG ORIG	28.480	41.359	102.329
AGG TRANS	35.152	25.355	167.969
MLE	50.620	58.087	43.565
TSCA	22.688	32.333	84.557
JS	22.687	27.958	49.336
LTJS	31.774	30.339	36.854

TABLE 11

COMBAT SUPPORT SMALL CELL FOM
($6 \leq$ INV ≤ 10)

	1981	1982	1983
	TRANSFORMED FOM		
1st LT			
AGG ORIG	0.252	0.113	0.136
AGG TRANS	0.203	0.151	0.136
MLE	0.155	0.102	0.040
TSCA	0.223	0.086	0.061
JS	0.196	0.082	0.057
LTJS	0.221	0.086	0.060
LTCOL			
AGG ORIG	0.112	0.143	0.302
AGG TRANS	0.353	0.494	0.616
MLE	0.166	0.162	0.141
TSCA	0.083	0.177	0.252
JS	0.068	0.215	0.290
LTJS	0.072	0.204	0.278
	ORIGINAL FOM		
1st LT			
AGG ORIG	9.745	9.120	8.899
AGG TRANS	32.916	9.111	15.834
MLE	24.051	15.102	6 459
TSCA	8.198	6.275	2.848
JS	8.697	6.188	3.580
LTJS	7.967	5.314	2.469
LTCOL			
AGG ORIG	5.572	4.976	2.419
AGG TRANS	9.841	11.651	12.791
MLE	4.665	6.313	4.904
TSCA	3.173	6.486	2.871
JS	3.397	5.871	5.177
LTJS	2.809	4.280	5.190

TABLE 12

GROUND COMBAT SMALL CELL FOM
(INV ≤ 5)

	1981	1982	1983
	TRANSFORMED FOM		
1st LT			
AGG ORIG	0.246	0.104	0.209
AGG TRANS	1.810	1.392	1.683
MLE	0.447	0.365	0.528
TSCA	0.245	0.161	0.285
JS	0.252	0.171	0.302
LTJS	0.253	0.170	0.300
LTCOL			
AGG ORIG	0.762	0.629	0.569
AGG TRANS	1.692	1.703	1.660
MLE	0.396	0.450	0.501
TSCA	0.380	0.388	0.459
JS	0.376	0.388	0.446
LTJS	0.377	0.385	0.451
	ORIGINAL FOM		
1st LT			
AGG ORIG	9.413	7.352	16.442
AGG TRANS	44.715	17.024	72.074
MLE	8.221	7.687	20.185
TSCA	10.589	8.740	17.879
JS	11.901	3.733	13.206
LTJS	9.775	3.916	11.794
LTCOL			
AGG ORIG	40.102	41.504	59.156
AGG TRANS	481.735	200.544	30543.860
MLE	24.230	15.215	24.940
TSCA	13.307	16.803	24.049
JS	18.610	16.513	25.658
LTJS	22.270	16.885	17.772

TABLE 13

GROUND COMBAT SMALL CELL FOM
($6 \leq INV \leq 10$)

	1981	1982	1983
	TRANSFORMED FOM		
1st LT			
AGG ORIG	0.097	0.150	0.160
AGG TRANS	0.054	0.115	0.131
MLE	0.011	0.180	0.251
TSCA	0.010	0.173	0.249
JS	0.012	0.163	0.237
LTJS	0.012	0.163	0.237
LTCOL			
AGG ORIG	0.530	1.114	0.394
AGG TRANS	0.224	0.696	0.204
MLE	0.295	0.462	0.600
TSCA	0.535	0.799	0.615
JS	0.509	0.777	0.578
LTJS	0.514	0.780	0.588
	ORIGINAL FOM		
1st LT			
AGG ORIG	5.659	3.658	5.081
AGG TRANS	3.490	2.767	4.614
MLE	2.809	5.387	13.237
TSCA	2.310	3.265	8.005
JS	2.501	3.970	8.155
LTJS	2.664	3.904	9.178
LTCOL			
AGG ORIG	9.408	47.989	10.095
AGG TRANS	5.452	43.834	8.123
MLE	6.041	13.789	13.300
TSCA	11.155	17.113	10.940
JS	9.481	18.416	11.942
LTJS	8.467	17.962	13.923

TABLE 14

AVIATION SMALL CELL PERCENTAGE OF TOTAL FOM
(INV ≤ 5)

	1981	1982	1983
		TRANSFORMED	
1st LT			
AGG ORIG	0.011	0.002	0.002
AGG TRANS	6.033	4.023	5.029
MLE	0.045	0.00001	0.008
TSCA	0.006	0.007	0.00002
JS	0.008	0.005	0.0002
LTJS	0.008	0.006	0.0002
LTCOL			
AGG ORIG	0.044	0.056	0.032
AGG TRANS	0.104	0.123	0.086
MLE	0.063	0.071	0.017
TSCA	0.045	0.056	0.032
JS	0.046	0.058	0.030
LTJS	0.046	0.057	0.030
		ORIGINAL	
1st LT			
AGG ORIG	0.168	0.005	0.003
AGG TRANS	0	0	0
MLE	0	0	0
TSCA	0.046	0.014	0.010
JS	0.065	0.010	0.008
LTJS	0.021	0.003	0.003
LTCOL			
AGG ORIG	0.081	0.115	0.068
AGG TRANS	0.005	0.560	0.984
MLE	0.170	0.149	0.072
TSCA	0.088	0.119	0.178
JS	0.138	0.151	0.137
LTJS	0.111	0.116	0.077

TABLE 15

AVIATION SMALL CELL PERCENTAGE OF TOTAL FOM
($6 \leq INV \leq 10$)

	1981	1982	1983
		TRANSFORMED	
1st LT			
AGG ORIG	0.021	0.014	0.00001
AGG TRANS	0.025	0.025	0.017
MLE	0.022	0.009	0.0004
TSCA	0.021	0.007	0.0008
JS	0.023	0.009	0.0004
LTJS	0.024	0.009	0.0004
LTCOL			
AGG ORIG	0.197	0.033	0.009
AGG TRANS	0.111	0.024	0.007
MLE	0.251	0.015	2E-6
TSCA	0.266	0.051	0.0001
JS	0.269	0.039	0.00003
LTJS	0.268	0.040	0.00003
		ORIGINAL	
1st LT			
AGG ORIG	0.050	0.047	0.006
AGG TRANS	0	0	0
MLE	0.028	0.014	0.010
TSCA	0.024	0.013	0.012
JS	0.029	0.016	0.010
LTJS	0.028	0.014	0.009
LTCOL			
AGG ORIG	0.490	0.084	0.007
AGG TRANS	0.991	0.162	0.0001
MLE	0.275	0.015	0.021
TSCA	0.403	0.072	0.024
JS	0.301	0.047	0.020
LTJS	0.227	0.034	0.013

TABLE 16

COMBAT SUPPORT SMALL CELL PERCENTAGE OF TOTAL FOM (INV ≤ 5)

	1981	1982	1983
		TRANSFORMED	
1st LT			
AGG ORIG	0.334	0.165	0.342
AGG TRANS	0.203	0.095	0.160
MLE	0.416	0.327	1.510
TSCA	0.316	0.184	0.430
JS	0.294	0.158	0.368
LTJS	0.312	0.176	0.409
LTCOL			
AGG ORIG	0.602	0.631	0.601
AGG TRANS	0.236	0.211	0.301
MLE	0.776	0.805	0.886
TSCA	0.828	0.823	0.844
JS	0.803	0.726	0.798
LTJS	0.806	0.782	0.812
		ORIGINAL	
1st LT			
AGG ORIG	0.446	0.354	0.561
AGG TRANS	0.347	0.151	0.997
MLE	0.222	0.105	0.234
TSCA	0.412	0.302	0.763
JS	0.388	0.235	0.511
LTJS	0.450	0.313	0.439
LTCOL			
AGG ORIG	0.822	0.848	0.977
AGG TRANS	0.721	0.567	0.929
MLE	0.884	0.867	0.899
TSCA	0.817	0.800	0.967
JS	0.816	0.761	0.905
LTJS	0.886	0.817	0.877

TABLE 17

COMBAT SUPPORT SMALL CELL PERCENTAGE OF TOTAL FOM
(6 ≤ INV ≤ 10)

	1981	1982	1983
		TRANSFORMED	
1st LT			
AGG ORIG	0.165	0.061	0.098
AGG TRANS	0.061	0.041	0.042
MLE	0.068	0.036	0.017
TSCA	0.161	0.043	0.041
JS	0.134	0.040	0.037
LTJS	0.155	0.042	0.041
LTCOL			
AGG ORIG	0.085	0.084	0.172
AGG TRANS	0.136	0.017	0.207
MLE	0.105	0.075	0.058
TSCA	0.091	0.105	0.144
JS	0.084	0.142	0.186
LTJS	0.087	0.131	0.173
		ORIGINAL	
1st LT			
AGG ORIG	0.123	0.123	0.098
AGG TRANS	0.209	0.070	0.0006
MLE	0.195	0.118	0 062
TSCA	0.112	0.078	0.023
JS	0.112	0.076	0.039
LTJS	0.099	0.064	0.036
LTCOL			
AGG ORIG	0.161	0.102	0.023
AGG TRANS	0.202	0.260	0.071
MLE	0.081	0.094	0.101
TSCA	0.114	0.161	0.033
JS	0.122	0.160	0.095
LTJS	0.078	0.115	0.123

TABLE 18

GROUND COMBAT SMALL CELL PERCENTAGE OF TOTAL FOM (INV ≤ 5)

	1981	1982	1983
		TRANSFORMED	
1st LT			
AGG ORIG	0.089	0.029	0.037
AGG TRANS	0.099	0.073	0.076
MLE	0.166	0.086	0.085
TSCA	0.125	0.047	0.054
JS	0.116	0.047	0.055
LTJS	0.124	0.049	0.050
LTCOL			
AGG ORIG	0.206	0.175	0.150
AGG TRANS	0.129	0.139	0.132
MLE	0.316	0.222	0.171
TSCA	0.248	0.161	0.138
JS	0.248	0.166	0.138
LTJS	0.240	0.163	0.138
		ORIGINAL	
1st LT			
AGG ORIG	0.119	0.092	0.164
AGG TRANS	0.115	0.067	0.224
MLE	0.073	0.053	0.118
TSCA	0.153	0.075	0.168
JS	0.143	0.038	0.113
LTJS	0.117	0.038	0.097
LTCOL			
AGG ORIG	0.400	0.349	0.533
AGG TRANS	0.814	0.586	0.996
MLE	0.574	0.350	0.440
TSCA	0.345	0.338	0.421
JS	0.438	0.326	0.417
LTJS	0.493	0.331	0.320

TABLE 19

GROUND COMBAT SMALL CELL PERCENTAGE OF TOTAL FOM
($6 \le INV \le 10$)

	1981	1982	1983
		TRANSFORMED	
1st LT			
AGG ORIG	0.035	0.042	0.028
AGG TRANS	0.003	0.006	0.006
MLE	0.004	0.042	0.041
TSCA	0.005	0.050	0.048
JS	0.005	0.045	0.043
LTJS	0.006	0.047	0.044
LTCOL			
AGG ORIG	0.144	0.310	0.104
AGG TRANS	0.017	0.057	0.016
MLE	0.236	0.228	0.205
TSCA	0.335	0.331	0.185
JS	0.322	0.333	0.179
LTJS	0.328	0.330	0.180
		ORIGINAL	
1st LT			
AGG ORIG	0.071	0.046	0.051
AGG TRANS	0.012	0.011	0.014
MLE	0.025	0.037	0.078
TSCA	0.003	0.035	0.075
JS	0.030	0.040	0.070
LTJS	0.031	0.038	0.076
LTCOL			
AGG ORIG	0.094	0.404	0.091
AGG TRANS	0.009	0.128	0.0003
MLE	0.143	0.317	0.235
TSCA	0.297	0.345	0.191
JS	0.223	0.363	0.194
LTJS	0.187	0.353	0.251

TABLE 20

AVIATION ATTRITION RATES FOR 1ST LTS

LOS	AGG ORIG	AGG TRANS	MLE	TSCA	JS	LTJS
0	0	0	0	0	0	0
1	0	0	0	0	0	0
2	.0485	.1079	.0150	.0160	.0167	.0167
3	.0485	.2218	.0265	.0277	.0291	.0286
4	.0485	.2466	.0304	.0310	.0325	.0318
5	.0485	.2155	.0954	.0901	.0914	.0914
6	.0485	.1367	.0488	.0490	.0498	.0498
7	.0485	.0687	.0600	.0609	.0610	.0610
8	.0485	.0037	.0724	.0654	.0641	.0641
9	.0485	.0039	.0443	.0474	.0465	.0465
10	.0485	.0043	.0659	.0704	.0691	.0691
11	.0485	.0255	.0861	.0871	.0838	.0838
12	.0485	0	.0727	.0812	.0723	.0724
13	.0485	0	0	.1433	.1139	.1193
14	.0485	0	0	.3260	.2622	.2780
15	0	0	0	0	0	0
16	0	0	0	0	0	0
17	0	0	0	0	0	0
18	0	0	0	0	0	0
19	0	0	0	0	0	0
20	0	0	0	0	0	0
21	0	0	0	0	0	0
22	0	0	0	0	0	0
23	0	0	0	0	0	0
24	0	0	0	0	0	0
25	0	0	0	0	0	0
26	0	0	0	0	0	0
27	0	0	0	0	0	0
28	0	0	0	0	0	0
29	0	0	0	0	0	0
30	0	0	0	0	0	0

TABLE 21

AVIATION ATTRITION RATES FOR LTCOLS

LOS	AGG ORIG	AGG TRANS	MLE	TSCA	JS	LTJS
0	0	0	0	0	0	0
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	0	0	0	0	0	0
4	0	0	0	0	0	0
5	0	0	0	0	0	0
6	0	0	0	0	0	0
7	0	0	0	0	0	0
8	0	0	0	0	0	0
9	0	0	0	0	0	0
10	0	0	0	0	0	0
11	0	0	0	0	0	0
12	0	0	0	0	0	0
13	0	0	0	0	0	0
14	0	0	0	0	0	0
15	.1457	0	0	.2694	.2345	.2438
16	.1457	.0596	.0952	.2517	.2305	.2343
17	.1457	.0861	0	.2370	.0251	.0251
18	.1457	.1948	0	.0084	.0106	.0096
19	.1457	.2444	.0055	.0102	.0131	.0113
20	.1457	.2669	.1158	.1221	.1260	.1252
21	.1457	.2726	.0928	.1005	.1050	.1033
22	.1457	.2788	.1670	.1697	.1729	.1729
23	.1457	.2667	.2098	.2118	.2135	.2135
24	.1457	.2352	.2012	.2087	.2096	.2096
25	.1457	.2019	.2568	.2622	.2602	.2602
26	.1457	.1268	.2877	.3038	.2973	.2973
27	.1457	.0384	.3288	.3368	.3241	.3260
28	.1457	.0003	.3415	.3303	.3123	.3161
29	.1457	.0199	.2963	.3078	.2864	.2911
30	.1457	.0244	.5385	.5251	.4940	.5066

TABLE 22

COMBAT SUPPORT ATTRITION RATES FOR 1ST LTS
CODE 07 ENGINEERS

LOS	AGG ORIG	AGG TRANS	MLE	TSCA	JS	LTJS
0	0	0	0	0	0	0
1	0	0	0	0	0	0
2	.2267	.3593	.2111	.1706	.2035	.1776
3	.2267	.3989	.2465	.2551	.2817	.2609
4	.2267	.3957	.1970	.2057	.2396	.2112
5	.2267	.3812	.2266	.2257	.2540	.2322
6	.2267	.3534	.3030	.3112	.3193	.3193
7	.2267	.3007	.3059	.3304	.3245	.3245
8	.2267	.1799	.4444	.4945	.4282	.4721
9	.2267	.1958	.1935	.2290	.2224	.2224
10	.2267	.2064	.1765	.2128	.2115	.2115
11	.2267	.1995	.1250	.1646	.1712	.1712
12	.2267	.1755	.0769	.1196	.1298	.1298
13	.2267	.1506	0	.0775	.0901	.0901
14	.2267	.1755	0	.0700	.0873	.0821
15	.2267	.1661	0	.0702	.0861	.0827
16	.2267	.1263	0	.0891	.0959	.0959
17	.2267	.1194	0	.0910	.0987	.0987
18	.2267	.1506	0	.0760	.0887	.0887
19	.2267	.1263	0	.0891	.0959	.0959
20	.2267	.1047	.1429	.1972	.1773	.1773
21	.2267	.1122	.1333	.2050	.1852	.1852
22	.2267	.0967	0	.1151	.1114	.1114
23	.2267	.0703	0	.1286	.1161	.1161
24	.2267	.0196	0	.2507	.1889	.2155
25	.2267	.0001	0	.3260	.2203	.2793
26	.2267	.0001	0	.3260	.2203	.2793
27	0	0	0	0	0	0
28	0	0	0	0	0	0
29	0	0	0	0	0	0
30	0	0	0	0	0	0

TABLE 23

COMBAT SUPPORT ATTRITION RATES FOR 1ST LTS
CODE 13 COMMUNICATIONS

LOS	AGG ORIG	AGG TRANS	MLE	TSCA	JS	LTJS
0	0	0	0	0	0	0
1	0	0	0	0	0	0
2	.2267	.3601	.2418	.1977	.2268	.205
3	.2267	.4080	.2910	.2846	.3078	.2901
4	.2267	.4046	.2687	.2664	.2921	.2719
5	.2267	.3910	.3399	.3417	.3511	.3484
6	.2267	.3563	.2558	.2673	.2841	.2757
7	.2267	.2963	.4198	.4319	.4048	.4186
8	.2267	.2879	.2432	.2589	.2645	.2645
9	.2267	.3257	.2281	.2335	.2507	.2433
10	.2267	.3330	.1120	.1132	.1489	.1203
11	.2267	.2940	.0760	.1015	.1326	.1100
12	.2267	.2218	.0513	.0920	.1135	.1033
13	.2267	.1122	.1333	.1922	.1753	.1753
14	.2267	.0403	0	.1648	.1348	.1379
15	.2267	.0001	0	.3629	.2467	.3147
16	0	0	0	0	0	0
17	0	0	0	0	0	0
18	0	0	0	0	0	0
19	.2267	.0001	0	.3260	.2203	.2793
20	.2267	.0001	0	.3260	.2203	.2793
21	0	0	0	0	0	0
22	0	0	0	0	0	0
23	0	0	0	0	0	0
24	0	0	0	0	0	0
25	0	0	0	0	0	0
26	0	0	0	0	0	0
27	0	0	0	0	0	0
28	0	0	0	0	0	0
29	0	0	0	0	0	0
30	0	0	0	0	0	0

TABLE 24

COMBAT SUPPORT ATTRITION RATES FOR 1ST LTS
CODE 20 MOTOR TRANSPORT

LOS	AGG ORIG	AGG TRANS	MLE	TSCA	JS	LTJS
0	0	0	0	0	0	0
1	0	0	0	0	0	0
2	.2267	.1661	0	.0878	.1014	.1014
3	.2267	.2508	.3137	.4064	.3747	.3899
4	.2267	.2724	.2857	.2693	.2699	.2699
5	.2267	.2784	.1791	.2207	.2316	.2316
6	.2267	.2189	.2632	.3449	.3191	.3267
7	.2267	.0796	.1818	.3036	.2517	.2744
8	.2267	.0299	1	.8007	.6388	.7683
9	.2267	.0607	.6667	.6184	.4918	.5845
10	.2267	.0403	0	.1866	.151	.1582
11	.2267	.1047	.1429	.1935	.1745	.1745
12	.2267	.1047	0	.1094	.1084	.1084
13	.2267	.1506	0	.0810	.0931	.0931
14	.2267	.1449	0	.0897	.0996	.0996
15	.2267	.1841	0	.0651	.0841	.0764
16	.2267	.1390	0	.0816	.0917	.0917
17	.2267	.0884	0	.1146	.1093	.1093
18	.2267	.0703	0	.1343	.1204	.1204
19	.2267	.0796	0	.1293	.1188	.1188
20	.2267	.0507	.2500	.3191	.2535	.2860
21	.2267	.0703	0	.1343	.1204	.1204
22	.2267	.0403	0	.2273	.1811	.1966
23	.2267	.0403	0	.1866	.1510	.1582
24	.2267	.0103	0	.2748	.2004	.2364
25	.2267	.0103	0	.2687	.1960	.2305
26	.2267	.0103	0	.2687	.1960	.2305
27	.2267	.0029	0	.2632	.1848	.2227
28	.2267	.0063	0	.4033	.2631	.3492
29	.2267	.0063	0	.4033	.2631	.3492
30	.2267	.0103	0	.2394	.1749	.2028

TABLE 25

COMBAT SUPPORT ATTRITION RATES FOR LTCOLS
CODE 07 ENGINEERS

LOS	AGG ORIG	AGG TRANS	MLE	TSCA	JS	LTJS
0	0	0	0	0	0	0
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	0	0	0	0	0	0
4	0	0	0	0	0	0
5	0	0	0	0	0	0
6	0	0	0	0	0	0
7	0	0	0	0	0	0
8	0	0	0	0	0	0
9	0	0	0	0	0	0
10	0	0	0	0	0	0
11	0	0	0	0	0	0
12	0	0	0	0	0	0
13	0	0	0	0	0	0
14	0	0	0	0	0	0
15	.1486	.0253	0	.3629	.2847	.2847
16	0	0	0	0	0	0
17	.1486	.1767	0	.0967	.1090	.1090
18	.1486	.2900	0	.0392	.0675	.0531
19	.1486	.3117	0	.0323	.0620	.0436
20	.1486	.3184	.2500	.2413	.2541	.2541
21	.1486	.3158	.0645	.0888	.1197	.1060
22	.1486	.3088	.1404	.1387	.1643	.1601
23	.1486	.2860	.0909	.1171	.1419	.1397
24	.1486	.2586	.2424	.2597	.2595	.2595
25	.1486	.2219	.2609	.2922	.2797	.2797
26	.1486	.1244	0	.1433	.1400	.1400
27	.1486	.1132	.7500	.6913	.5861	.6272
28	.1486	.0420	.6667	.5462	.4389	.4577
29	.1486	.0420	0	.2954	.2394	.2394
30	.1486	.1010	.5714	.4929	.4150	.4224

TABLE 26

COMBAT SUPPORT ATTRITION RATES FOR LTCOLS
CODE 13 COMMUNICATIONS

LOS	AGG ORIG	AGG TRANS	MLE	TSCA	JS	LTJS
0	0	0	0	0	0	0
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	0	0	0	0	0	0
4	0	0	0	0	0	0
5	0	0	0	0	0	0
6	0	0	0	0	0	0
7	0	0	0	0	0	0
8	0	0	0	0	0	0
9	0	0	0	0	0	0
10	0	0	0	0	0	0
11	0	0	0	0	0	0
12	0	0	0	0	0	0
13	0	0	0	0	0	0
14	0	0	0	0	0	0
15	0	0	0	0	0	0
16	.1486	.01	0	.4033	.3067	.3067
17	.1486	.2018	0	.1091	.1233	.1233
18	.1486	.2817	0	.0445	.0723	.0598
19	.1486	.3221	0	.02955	.0596	.0396
20	.1486	.3378	.1463	.1762	.2014	.1959
21	.1486	.3330	.1299	.1713	.1964	.1913
22	.1486	.3330	.1558	.1910	.2135	.2119
23	.1486	.3026	.1509	.1491	.1726	.1719
24	.1486	.2817	.2857	.2577	.2617	.2617
25	.1486	.2524	.3226	.2887	.2823	.2823
26	.1486	.1900	.2353	.2658	.2522	.2522
27	.1486	.1444	.3636	.3866	.3401	.3401
28	.1486	.1132	0	.1775	.1656	.1656
29	.1486	.1132	.2500	.3483	.3019	.3019
30	.1486	.2586	.3636	.3884	.3652	.3652

TABLE 27

COMBAT SUPPORT ATTRITION RATES FOR LTCOLS
CODE 20 MOTOR TRANSPORT

LOS	AGG ORIG	AGG TRANS	MLE	TSCA	JS	LTJS
0	0	0	0	0	0	0
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	0	0	0	0	0	0
4	0	0	0	0	0	0
5	0	0	0	0	0	0
6	0	0	0	0	0	0
7	0	0	0	0	0	0
8	0	0	0	0	0	0
9	0	0	0	0	0	0
10	0	0	0	0	0	0
11	0	0	0	0	0	0
12	0	0	0	0	0	0
13	0	0	0	0	0	0
14	0	0	0	0	0	0
15	0	0	0	0	0	0
16	.1486	.0100	0	.4033	.3067	.3067
17	.1486	.0100	0	.4033	.3067	.3067
18	.1486	.0253	0	.3260	.2562	.2562
19	.1486	.0558	0	.2113	.1794	.1794
20	.1486	.0878	.3333	.3641	.3078	.3078
21	.1486	.0736	.4000	.4277	.3542	.3542
22	.1486	.0736	.4000	.4277	.3542	.3542
23	.1486	.0100	0	.4033	.3067	.3067
24	0	0	0	0	0	0
25	0	0	0	0	0	0
26	0	0	0	0	0	0
27	0	0	0	0	0	0
28	0	0	0	0	0	0
29	.1486	.0253	0	.3260	.2562	.2562
30	.1486	.0878	.6667	.6067	.5057	.5333

TABLE 28

GROUND COMBAT ATTRITION RATES FOR 1ST LTS
CODE 03 INFANTRY

LOS	AGG ORIG	AGG TRANS	MLE	TSCA	JS	LTJS
0	0	0	0	0	0	0
1	.1991	0	0	.4033	.2713	.3177
2	.1991	.3347	.1788	.1382	.1514	.1438
3	.1991	.3814	.1935	.1899	.2034	.1944
4	.1991	.3815	.2165	.2148	.2269	.2195
5	.1991	.3709	.1746	.1748	.1885	.1796
6	.1991	.3434	.2604	.2581	.2646	.2646
7	.1991	.2973	.2194	.2250	.2305	.2305
8	.1991	.2283	.1463	.1497	.1554	.1554
9	.1991	.2626	.1511	.1548	.1626	.1626
10	.1991	.2622	.1516	.1611	.1685	.1685
11	.1991	.2241	.0808	.0920	.1006	.1002
12	.1991	.1013	.1039	.1240	.1222	.1222
13	.1991	.0186	.0541	.0928	.0851	.0850
14	.1991	.0626	0	.1179	.0887	.0887
15	.1991	.0626	.1667	.2294	.1811	.1889
16	.1991	.2026	0	.1913	.1372	.1463
17	0	0	0	0	0	0
18	0	0	0	0	0	0
19	0	0	0	0	0	0
20	0	0	0	0	0	0
21	0	0	0	0	0	0
22	0	0	0	0	0	0
23	0	0	0	0	0	0
24	0	0	0	0	0	0
25	0	0	0	0	0	0
26	0	0	0	0	0	0
27	0	0	0	0	0	0
28	0	0	0	0	0	0
29	0	0	0	0	0	0
30	0	0	0	0	0	0

TABLE 29

GROUND COMBAT ATTRITION RATES FOR 1ST LTS
CODE 05 ARTILLERY

LOS	AGG ORIG	AGG TRANS	MLE	TSCA	JS	LTJS
0	0	0	0	0	0	0
1	0	0	0	0	0	0
2	.1991	.2888	.1170	.1050	.1168	.1115
3	.1991	.3408	.2052	.2096	.2193	.2160
4	.1991	.3423	.2462	.2480	.2552	.2547
5	.1991	.3197	.1948	.1992	.2080	.2063
6	.1991	.2724	.3016	.3049	.3023	.3023
7	.1991	.1671	.3333	.3329	.3184	.3184
8	.1991	.1294	.1263	.1323	.1321	.1321
9	.1991	.1390	.2353	.2452	.2360	.2360
10	.1991	.1503	.0901	.0991	.1029	.1029
11	.1991	.0888	.0571	.0783	.0791	.0791
12	.1991	.0342	.0454	.0851	.0803	.0803
13	.1991	.0039	.1000	.1609	.1341	.1341
14	.1991	.2538	0	.1758	.1216	.1305
15	.1991	0	0	.3260	.2189	.2523
16	.1991	0	0	.3260	.2189	.2523
17	0	0	0	0	0	0
18	0	0	0	0	0	0
19	0	0	0	0	0	0
20	0	0	0	0	0	0
21	0	0	0	0	0	0
22	0	0	0	0	0	0
23	0	0	0	0	0	0
24	0	0	0	0	0	0
25	0	0	0	0	0	0
26	0	0	0	0	0	0
27	0	0	0	0	0	0
28	0	0	0	0	0	0
29	0	0	0	0	0	0
30	0	0	0	0	0	0

TABLE 30

GROUND COMBAT ATTRITION RATES FOR 1ST LTS
CODE 10 TANKS

LOS	AGG ORIG	AGG TRANS	MLE	TSCA	JS	LTJS
0	0	0	0	0	0	0
1	0	0	0	0	0	0
2	.1991	.1573	.2906	.2368	.2300	.2300
3	.1991	.2506	.2490	.2505	.2506	.2506
4	.1991	.2515	.2390	.2435	.2441	.2441
5	.1991	.2488	.1633	.1697	.1756	.1756
6	.1991	.1987	.3230	.3216	.3112	.3112
7	.1991	.1030	.3333	.3449	.3223	.3241
8	.1991	.0186	.1081	.1377	.1246	.1246
9	.1991	.0364	.1778	.2113	.1931	.1931
10	.1991	.0651	.1034	.1167	.1121	.1121
11	.1991	.0207	.0526	.0855	.0789	.0789
12	.1991	.0801	0	.1498	.1131	.1149
13	.1991	0	0	.1927	.1311	.1431
14	.1991	0	0	.4033	.2713	.3177
15	0	0	0	0	0	0
16	0	0	0	0	0	0
17	.1991	0	0	.3260	.2189	.2523
18	0	0	0	0	0	0
19	0	0	0	0	0	0
20	0	0	0	0	0	0
21	0	0	0	0	0	0
22	0	0	0	0	0	0
23	0	0	0	0	0	0
24	0	0	0	0	0	0
25	0	0	0	0	0	0
26	0	0	0	0	0	0
27	0	0	0	0	0	0
28	0	0	0	0	0	0
29	0	0	0	0	0	0
30	0	0	0	0	0	0

TABLE 31

GROUND COMBAT ATTRITION RATES FOR LTCOLS
CODE 03 INFANTRY

LOS	AGG ORIG	AGG TRANS	MLE	TSCA	JS	LTJS
0	0	0	0	0	0	0
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	0	0	0	0	0	0
4	0	0	0	0	0	0
5	0	0	0	0	0	0
6	0	0	0	0	0	0
7	0	0	0	0	0	0
8	0	0	0	0	0	0
9	0	0	0	0	0	0
10	0	0	0	0	0	0
11	0	0	0	0	0	0
12	0	0	0	0	0	0
13	0	0	0	0	0	0
14	0	0	0	0	0	0
15	.1278	.1361	0	.3002	.2664	.2749
16	.1278	.0249	0	.1000	.0962	.0962
17	.1278	.2097	0	.0173	.0210	.0193
18	.1278	.3101	0	.0062	.0098	.0070
19	.1278	.3341	.0095	.0139	.0188	.0148
20	.1278	.3431	.1224	.1273	.1342	.1299
21	.1278	.3395	.0752	.0804	.0877	.0826
22	.1278	.3328	.1253	.1285	.1351	.1313
23	.1278	.3155	.1365	.1413	.1471	.1445
24	.1278	.3001	.2113	.2171	.2202	.2202
25	.1278	.2658	.2090	.2163	.2182	.2182
26	.1278	.2236	.2464	.2536	.2524	.2524
27	.1278	.1703	.2889	.2964	.2911	.2911
28	.1278	.0984	.2308	.2343	.2281	.2281
29	.1278	.0744	.2326	.2111	.2047	.2047
30	.1278	.1550	.4500	.4551	.4421	.4460

TABLE 32

GROUND COMBAT ATTRITION RATES FOR LTCOLS
CODE 05 ARTILLERY

LOS	AGG ORIG	AGG TRANS	MLE	TSCA	JS	LTJS
0	0	0	0	0	0	0
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	0	0	0	0	0	0
4	0	0	0	0	0	0
5	0	0	0	0	0	0
6	0	0	0	0	0	0
7	0	0	0	0	0	0
8	0	0	0	0	0	0
9	0	0	0	0	0	0
10	0	0	0	0	0	0
11	0	0	0	0	0	0
12	0	0	0	0	0	0
13	0	0	0	0	0	0
14	0	0	0	0	0	0
15	0	0	0	0	0	0
16	.1278	.0215	0	.1994	.1807	.1818
17	.1278	.1255	0	.0314	.0340	.0340
18	.1278	.2219	0	.0141	.0177	.0158
19	.1278	.2496	.0347	.0477	.0528	.0504
20	.1278	.2463	.2500	.2530	.2527	.2527
21	.1278	.2357	.0654	.0747	.0795	.0782
22	.1278	.2201	.1045	.0914	.0956	.0956
23	.1278	.1826	.1414	.1622	.1630	.1630
24	.1278	.1533	.3544	.3577	.3487	.3488
25	.1278	.0984	.1923	.2046	.1999	.1999
26	.1278	.0597	.5263	.5180	.4962	.5050
27	.1278	.0329	.6667	.5840	.5432	.5610
28	.1278	.0215	.4000	.4096	.3774	.3876
29	.1278	.1361	.8000	.6480	.5981	.6210
30	.1278	.0133	.7273	.6592	.6192	.6385

TABLE 33

GROUND COMBAT ATTRITION RATES FOR LTCOLS
CODE 10 TANKS

LOS	AGG ORIG	AGG TRANS	MLE	TSCA	JS	LTJS
0	0	0	0	0	0	0
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	0	0	0	0	0	0
4	0	0	0	0	0	0
5	0	0	0	0	0	0
6	0	0	0	0	0	0
7	0	0	0	0	0	0
8	0	0	0	0	0	0
9	0	0	0	0	0	0
10	0	0	0	0	0	0
11	0	0	0	0	0	0
12	0	0	0	0	0	0
13	0	0	0	0	0	0
14	0	0	0	0	0	0
15	0	0	0	0	0	0
16	.1278	0	0	.3260	.2811	.2943
17	.1278	.0037	0	.1354	.1239	.1239
18	.1278	.0129	0	.0695	.0665	.0665
19	.1278	.0716	0	.0427	.0437	.0437
20	.1278	.0744	.1860	.1828	.1778	.1778
21	.1278	.0628	.0513	.0861	.0851	.0851
22	.1278	.0773	.1818	.2047	.1988	.1988
23	.1278	.0567	.0541	.0883	.0870	.0870
24	.1278	.0505	.2857	.2982	.2857	.2860
25	.1278	.0129	.3478	.4256	.4026	.4096
26	.1278	.0002	.1333	.2135	.1978	.1980
27	.1278	.0037	.3077	.3171	.2933	.2983
28	.1278	.0329	.2222	.2630	.2385	.2428
29	.1278	.0699	.2857	.3350	.3027	.3113
30	.1278	.0001	.3750	.3841	.3590	.3659

V. CONCLUSIONS AND RECOMMENDATIONS

A. CONCLUSIONS

This study investigated the performance of various attrition rate estimation schemes, with particular emphasis on small cell performance. The hope was to identify a specific method, the limited translation James-Stein technique, as the best predictor of cell rates. This has not been done. In fact, the relative performance of four schemes (MLE, TSCA, James-Stein, limited translation James-Stein) was so varied that no clear winner or pattern of performance emerged.

Several points are of interest. First, even though the performance of the four techniques listed above varied, they were all uniformly better than either original or transformed scale aggregate estimation. Thus four excellent candidates for replacing the present attrition rate estimation methods are available for further testing.

Second, the models seem very sensitive to small changes in parameters. When investigating the behavior of the Freeman-Tukey transform, and James-Stein and limited translation James-Stein estimation, several choices had to be made concerning the methods used to calculate various quantities, e.g., the grand mean, maximum likelihood estimators, and the inventories. Large changes in FOM values and cell attrition rates were observed as different methods were tried. This lack of robustness is troublesome, and indicates the models are not ready for implementation. We feel a better method of aggregating the OF/LOS/grade cells would do much to relieve this problem.

Third, in the small cells, the Freeman-Tukey transformation fails to normalize the cell means or stabilize the variance. This failure is an inherent aspect of dealing with

small cells. In general, the research effort must be alert to finding alternative ways to manage the small cell problem.

B. RECOMMENDATIONS

At present, no method investigated here is recommended for implementation. Further study in the following areas is needed.

1. Aggregation. The work of Amin Elseramegy [Ref. 2] needs to be carried forward to identify statistically well behaved aggregates.

2. TSCA. The performance of the TSCA scheme should be investigated in light of its surprising performance.

3. Yearly Update. The problem of when and how to update the estimation data base must be solved.

4. Use of Different Estimators. Investigate the use of several different estimators (such as MLE, TSCA, James-Stein, and limited translation James-Stein), either in combination to yield average cell estimates, or separately to estimate the rates of different cells based on OF, military occupation specialty (MOS) as a subset of OF, LOS, grade, or desired estimation year. This would require the aggregation problem solved.

5. Robust Parametric Empirical Bayes. Investigate the use of the robust parametric empirical Bayes model to determine global and small cell estimation efficacy.

APPENDIX A
ESTIMATION ALGORITHMS

1. JAMES-STEIN ALGORITHM

As used by Major Tucker [Ref. 1], this algorithm calculates the James-Stein estimator of attrition rates. The following formulas use a double indexing system, (i,j), to identify cells in an aggregate and use $t = 1,\ldots,n$ to identify time periods. This usage is different from that of the main text which used the single subscript $i = 1,\ldots,k$ to identify cells. This was convenient for purposes of ANOVA. The current double indexing is needed to access the real data in its natural form.

Notation:

I = number of LOS cells in the chosen aggregate

J = number of OF cells in the chosen aggregate

$INV_{ij}(t)$ = inventory with LOS=i and OF=j at beginning of year t, $t=1,\ldots,T$

$y_{ij}(t)$ = number of attritions in cell (i,j) during year t

$n_{ij}(t) = \text{maximum } \{y_{ij}(t), .5[INV_{ij}(t)+INV_{ij}(t+1)]\}$

D = incidence matrix which identifies cells with sampling zeros (average inventory zero for all estimation years)

$D_{ij} = 0$ if cell is a structural zero

$D_{ij} = 1$ if cell is not a structural zero

STEP 1: Use a variance stabilizing transformation (Freeman-Tukey).

$$x_{ij}(t) = 0.5[n_{ij}(t)+0.5]^{\frac{1}{2}}\{\sin^{-1}[2(y_{ij}(t)/(n_{ij}(t)+1)-1] + \sin^{-1}[2(y_{ij}(t)+1)/(n_{ij}(t)+1)-1]\}. \tag{A.1}$$

STEP 2: Form the cell means and the grand mean.

$$\bar{X}_{ij} = (1/T)\sum_{t} X_{ij}(t), \qquad \forall\ i,j,t, \tag{A.2}$$

$$\bar{\bar{X}} = (1/K)\sum_{i}\sum_{j} X_{ij} D_{ij}, \qquad \forall\ i,j, \tag{A.3}$$

$$K = \sum_{i}\sum_{j} D_{ij}, \qquad \forall\ i,j. \tag{A.4}$$

STEP 3: Form the sum of squares error (SSE), and the sum of squares between (SSB), using the total sum of squares (SST).

$$SST = \sum_{i}\sum_{j}\sum_{t}[X_{ij}(t)-\bar{\bar{X}}]^2 D_{ij}, \qquad \forall\ i,j,t \tag{A.5}$$

$$SSE = \sum_{i}\sum_{j}\sum_{t}[X_{ij}(t)-\bar{X}_{ij}]^2 D_{ij}, \qquad \forall\ i,j,t \tag{A.6}$$

$$SSB = SST-SSE \tag{A.7}$$

STEP 4: Compute the set of James-Stein estimators in the transformed scale.

$$C_J = 1 - (K-3)SSE/(K(T-1)+2)SSB \tag{A.8}$$

$$P_J(i,j) = \begin{cases} \bar{\bar{X}} + C_J(\bar{X}_{ij}-\bar{\bar{X}}) & \text{if } D_{ij}=1 \\ \text{undefined} & \text{if } D_{ij}=0 \end{cases} \tag{A.9}$$

STEP 5: Invert the transform to produce the attrition rates r_{ij}. This corrects a typographical error that appeared in this step in reference 1.

$$n_{ij} = (1/T)\sum_{t} n_{ij}(t), \qquad \forall\ i \tag{A.10}$$

$$v_{ij} = P_J(i,j)/(n_{ij}+.5)^{\frac{1}{2}} \tag{A.11}$$

$$r = \begin{cases} 0 & v_{ij} \le -\pi/2 \\ .5[1+\sin y_{ij}] \quad \text{if} & -\pi/2 < v_{ij} < \pi/2" \\ 1 & v_{ij} \ge \pi/2 \end{cases} \tag{A.12}$$

2. LIMITED TRANSLATION JAMES-STEIN ALGORITHM

This algorithm calculates the limited translation James-Stein estimator of attrition rates. Because the only difference between this algorithm and the James-Stein algorithm of Appendix A, Section 1 is in STEP 4, only that changed step is presented. All other steps remain the same.

STEP 4: Compute the set of limited translation James-Stein estimators in the transformed scale. Choose d from the range $[0,\infty]$. Values between .2 and 1.0 seem to give the best results.

$$C_J = 1 - (K-3)SSE/(K(T-1)+2)SSB \tag{A.13}$$

$$A = (K(T-1)+2)SSB/(K-3)SSE - 1 \tag{A.14}$$

$$u = (\bar{X}_{ij} - \bar{\bar{X}})^2/(A+1) \tag{A.15}$$

$$\rho(u) = \text{minimum } (1, d/u^{\frac{1}{2}}) \tag{A.16}$$

$$C_{LJ} = 1 - \rho(u)[(K-3)SSE/(K(T-1)+2)SSB] \tag{A.17}$$

$$P_{LJ}(i,j) = \begin{cases} \bar{\bar{X}} + C_{LJ}(\bar{X}_{ij} - \bar{\bar{X}}) & \text{if } D_{ij}=1 \\ \text{undefined} & \phantom{\text{if }} D_{ij}=0 \end{cases} \tag{A.18}$$

APPENDIX B

UTILIZATION OF THE FREEMAN-TUKEY ARCSINE TRANSFORMATION

1. GENERAL

Limited translation James-Stein estimation as discussed in the Efron and Morris articles [Ref. 5,6] makes the following assumptions:

(1) The distribution of the number of losses is normally distributed.

(2) The variances of the normally distributed number of losses are equal.

Since the raw losses are not normally distributed, the Freeman-Tukey arcsine transformation

$$x = 0.5[n+0.5]^{\frac{1}{2}}\{\sin^{-1}[2y/(n+1)-1] + \sin^{-1}[2(y+1)/(n+1)-1]\} \qquad (B.1)$$

was used, given the central inventory n, to transform the raw losses y into transformed losses x.

2. NORMALITY OF TRANSFORMED LOSSES

Tucker [Ref. 1] demonstrated that the distribution of y given n is Binomial (p), where the parameter p is the probability of an individual loss. The probability mass function of this discrete distribution is

$$P\{Y=y\} = [n!/(y!(n-y)!)]\ p^{y}(1-p)^{n-y}. \qquad (B.2)$$

Since there is a unique mapping from Y to X, it is true that

$$P\{X=x\} = P\{Y=y\} \qquad (B.3)$$

if

$$x = f(y,n). \tag{B.4}$$

To check normality, a central inventory n was chosen. Possible losses y, from 0 to n, were identified. Then the above probability mass function and transformed losses x were calculated. This data was then displayed on an x versus P{X=x} graph. Figures B.1, B.2, and B.3 are graphs for n equals 5, 8, and 10. Probabilities of loss are for p equals .05, .1, .15, and .2. The ranges of greatest concern were those of low central inventory n and low probability of loss p.

By inspection, the higher the values of n and p, the better is the normal approximation. The Freeman-Tukey transformation is unreliable at low values of n and p.

3. VARIANCE STABILITY OF TRANSFORMED LOSSES

The mean and variance of the Binomial distribution is well known. Again using the fact that if x is a function of y and n, then

$$P\{X=x\} = P\{Y=y\}, \tag{B.5}$$

and the variance of x can be calculated, given values of n, y and x. The values of the variance of x were calculated, and are graphed in Figure B.4.

By inspection, once n equals 7, the variance stability at values of p less than .2 is poor. Again, the Freeman-Tukey transformation is unreliable at low values of n and p.

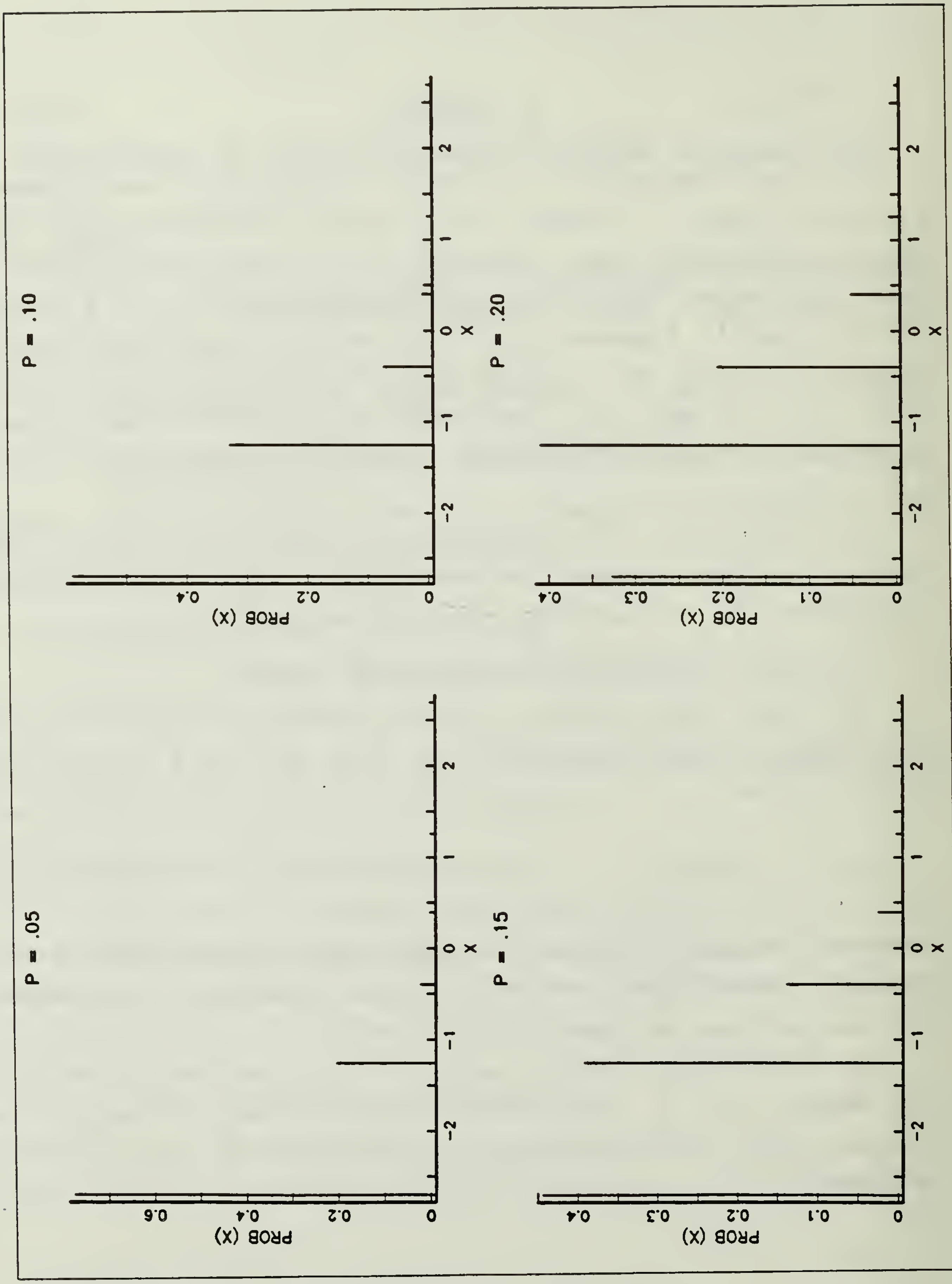

Figure B.1 Distribution of the Central Inventory N = 5

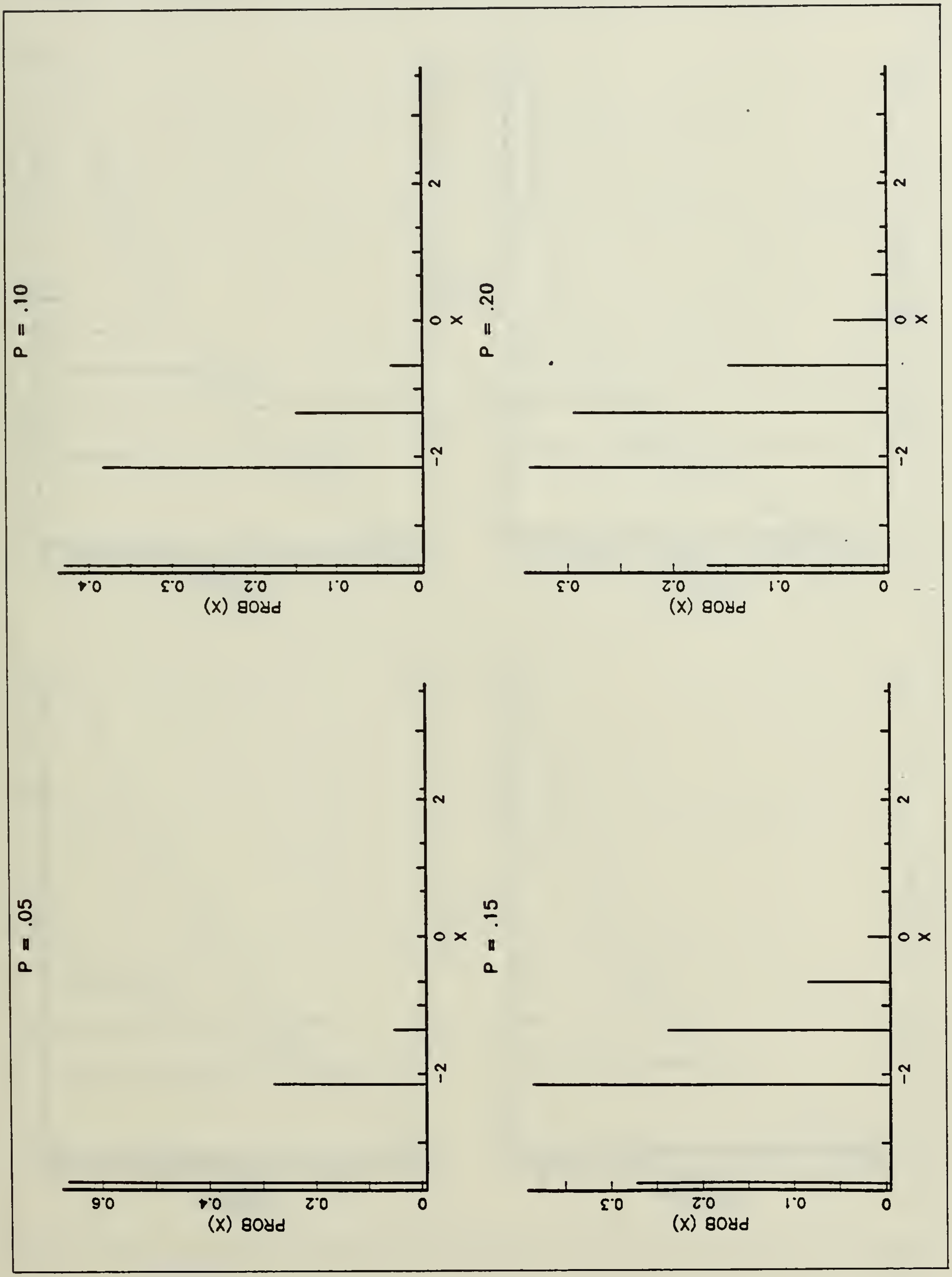

Figure B.2   Distribution of the Central Inventory   N = 8

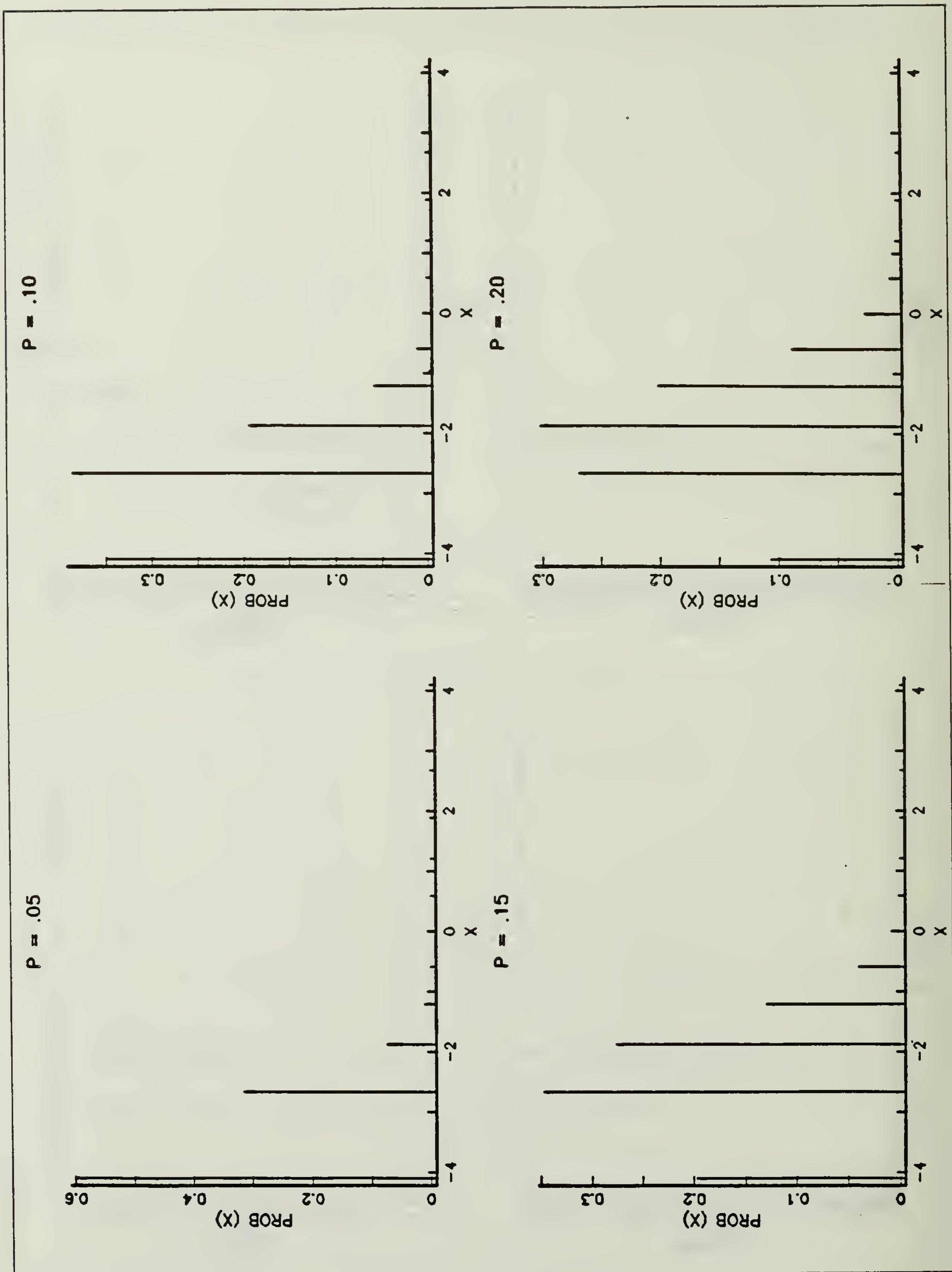

Figure B.3 Distribution of the Central inventory N = 10

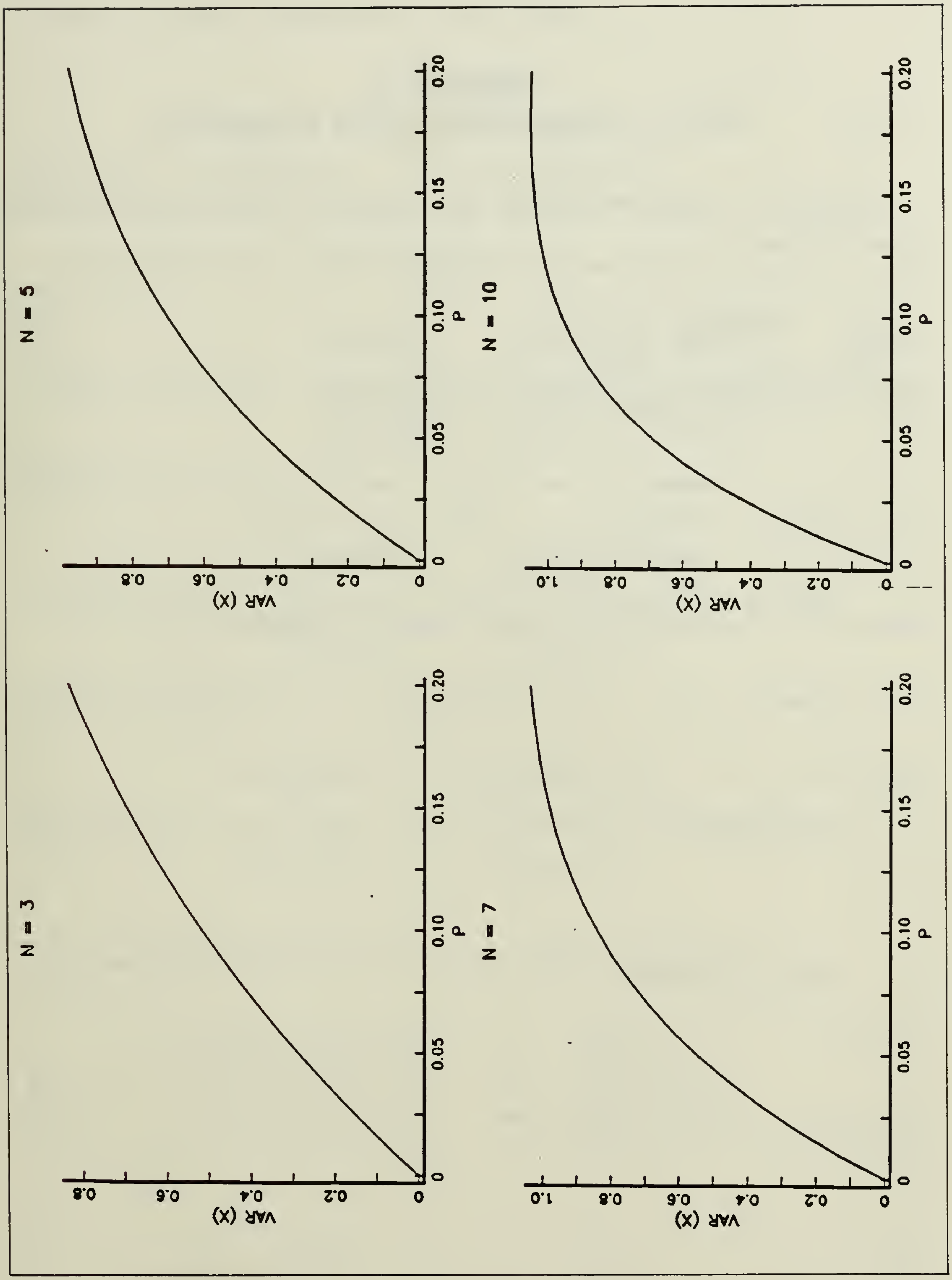

Figure B.4   Variance of the Central Inventory

APPENDIX C

LIMITED TRANSLATION FACTOR DERIVATION

This is a derivation of the form of the limited translation factor

$$\rho(u) = \text{minimum } (1, d/u^{\frac{1}{2}}) \tag{C.1}$$

Given: From Efron and Morris [Ref. 5],

$$\delta_{A,M}(x) = \begin{cases} x+M & \\ Ax/(A+1) & \text{if} \\ x-M & \end{cases} \quad \begin{matrix} x<-C \\ x\in[-C,C] \\ x>C \end{matrix} \tag{C.2}$$

where, if X_{ij} and $\bar{X}$ are those used in Appendix A,

$$C = M(A+1) \tag{C.3}$$

$$d = M(A+1)^{\frac{1}{2}} \tag{C.4}$$

$$u = (\bar{X}_{ij}-\bar{\bar{X}})^2/(A+1) \tag{C.5}$$

$$\delta_{A,M}(x) = [1-\rho(u)/(A+1)]x. \tag{C.6}$$

Derivation: From equation C.2 we have,

$$\delta_{A,M}(x) = \begin{cases} x+M & \\ Ax/(A+1) & \text{if} \\ x-M & \end{cases} \quad \begin{matrix} x<-C \\ x\in[-C,C] \\ x>C \end{matrix} \tag{C.7}$$

By substitution, and multiplication by x/x,

$$(1-\rho(u)/(A+1))x = \begin{cases} (x+M)x/x & \\ Ax/(A+1) & \text{if} \\ (x-M)x/x & \end{cases} \quad \begin{matrix} [u(A+1)]^{\frac{1}{2}}<-M(A+1) \\ [u(A+1)]^{\frac{1}{2}}\in[-M(A+1),M(A+1)] \\ [u(A+1)]^{\frac{1}{2}}>M(A+1) \end{matrix} \tag{C.8}$$

By multiplication by 1/x, and cancellation,

$$1-\rho(u)/(A+1) = \begin{cases} (x+M)x/x & \\ Ax/(A+1) & \text{if} \\ (x-M)x/x & \end{cases} \quad \begin{matrix} u^{\frac{1}{2}}<-M(A+1)^{\frac{1}{2}} \\ u^{\frac{1}{2}}\in[-M(A+1)^{\frac{1}{2}},M(A+1)^{\frac{1}{2}}] \\ u^{\frac{1}{2}}>M(A+1)^{\frac{1}{2}} \end{matrix} \tag{C.9}$$

By subtracting 1, and multiplying by -1,

$$\rho(u)/(A+1) = \begin{cases} 1-(x+M)/x & \\ 1-A/(A+1) & \text{if} \\ 1-(x-M)/x & \end{cases} \quad \begin{matrix} u^{\frac{1}{2}}<-M(A+1)^{\frac{1}{2}} \\ u^{\frac{1}{2}}\in[-M(A+1)^{\frac{1}{2}},M(A+1)^{\frac{1}{2}}] \\ u^{\frac{1}{2}}>M(A+1)^{\frac{1}{2}} \end{matrix} \tag{C.10}$$

By multiplying by (A+1), and substitution,

$$\rho(u) = \begin{cases} (A+1)[1-(x+M)/x] & \\ (A+1)[1-A/(A+1)] & \text{if} \\ (A+1)[1-(x-M)/x] & \end{cases} \quad \begin{matrix} u^{\frac{1}{2}}<-d \\ u^{\frac{1}{2}}\in[-d,d] \\ u^{\frac{1}{2}}>d \end{matrix} \tag{C.11}$$

$$= \begin{cases} -M((A+1)/u)^{\frac{1}{2}} & \\ 1 & \text{if} \\ M((A+1)/u)^{\frac{1}{2}} & \end{cases} \quad \begin{matrix} u^{\frac{1}{2}}<-d \\ u^{\frac{1}{2}}\in[-d,d] \\ u^{\frac{1}{2}}>d \end{matrix} \tag{C.12}$$

$$
= \begin{cases} -d/u^{\frac{1}{2}} & \text{if } u^{\frac{1}{2}}<-d \\ 1 & \text{if } u^{\frac{1}{2}}\in[-d,d] \\ d/u^{\frac{1}{2}} & \text{if } u^{\frac{1}{2}}>d \end{cases} \tag{C.13}
$$

If $u^{\frac{1}{2}}<-d$, then $0<-d/u^{\frac{1}{2}}<1$, and if $u^{\frac{1}{2}}>d$, then $0<d/u^{\frac{1}{2}}<1$. Therefore, the last equation may be written as

$$
\rho(u) = \text{minimum } (1,d/u^{\frac{1}{2}}). \tag{C.14}
$$

It is worth noting that if $x\in[-C,C]$ and $\rho(u)=1$, then

$$
\delta_{A,M}(x) = Ax/(A+1). \tag{C.15}
$$

Also, if $x\notin[-C,C]$ and $0<\rho(u)<1$, then

$$
Ax/(A+1) < \delta_{A,M}(x) < x. \tag{C.16}
$$

Thus, if x is outside the interval [-C,C], then this x is translated less than any x's inside the interval [-C,C]. This confirms the graphical representation of limited translation shown in Figure 3.1.

APPENDIX D
DATA MANIPULATION

1. GENERAL

This appendix documents the steps and programs used to take the raw data from magnetic tape to the output of figures of merit and attrition rates. The program languages include JCL,WATFIV, and APL. The author is familiar with the system at the Naval Postgraduate School, and the instructions are therefore specific for that system. However, minor changes should be all that are necessary to implement these procedures elsewhere. To achieve good results, the procedures should be followed in the order presented.

2. CONVERSION OF RAW DATA FROM TAPE TO APL

The original data is on a magnetic tape named COUNTS, prepared by NPRDC. The tape is held by either Professor R.R. Read, or by the Computer Center in Professor Read's name. Ensure the tape is properly logged into the Computer Center, and submit the JCL program IEBGENER in Figure D.1 to put the tape on mass storage. The data set in mass storage should be named MSS.SXXXX.COUNTS, where XXXX is the user ID number of the operator.

Once in mass storage, submit the JCL program MSSCOUN in Figure D.2 to move the data from mass storage to the MVS004 disk. Data on this disk is accessible from CMS.

Use the system exec GETMVS to move the data from the MVS004 disk to your disk. Simply enter 'GETMVS' on the computer and follow the directions. The identification requested by the prompts of GETMVS will be 'SXXXX COUNTS'. Since the data set is large (16,093 lines of 53 columns each), it is advisable to get additional workspace by either applying for a B disk of at least 8 cylinders, or by getting

```
//  EXEC PGM=IEBGENER
//SYSPRINT DD SYSOUT=A
//SYSIN   DD DUMMY
//SYSUT1 DD UNIT=3400-5,VOL=SER=COUNTS,DISP=(OLD,PASS),
//            DCB=(RECFM=FB,LRECL=53,BLKSIZE=21200,DEN=4),
//            LABEL=(1,SL,,IN),DSN=COUNTS
//SYSUT2 DD UNIT=3330V,MSVGP=PUB4B,DISP=(NEW,CATLG),
//            DCB=(RECFM=FB,LRECL=53,BLKSIZE=12985),
//            DSN=MSS.S1662.COUNTS
//
```

Figure D.1 JCL Program

```
//COUNTS JOB (1662,9999),'MAJ J.R.ROBINSON',CLASS=B
// EXEC PGM=IEBGENER
//SYSPRINT DD SYSOUT=A
//SYSIN DD DUMMY
//SYSUT1 DD DSN=MSS.S1662.COUNTS,DISP=SHR
//SYSUT2 DD DSN=S1662.COUNTS,VOL=SER=MVS004,UNIT=3350,
//        SPACE=(CYL,(2,2),RLSE),DISP=(NEW,KEEP)
//
```

Figure D.2 JCL Program MSSCOUN

a temporary C disk of at least 8 cylinders. Be aware that a C disk disappears once logged off, and all data on it is erased.

The WATFIV program SORT in Figure D.3 should now be used to separate the data in COUNTS into seven files, one file for each fiscal year. The data files can be conveniently named COUNXX DATA, where XX is the year, e.g., 77.

Using the WATFIV program INV in Figure D.4 , create an array of inventory indices for each fiscal year. The program should read in the data sets prepared in step 4 above, e.g., COUN77 DATA. The output is read into a file that can be named INVXX DATA, where XX is the fiscal year, e.g., 77. Note that, for each fiscal year, a different DO loop is used, since the number of data rows vary form year-to-year.

```
$JOB
C
C     THIS PROGRAM ASSIGNS THE DATA FROM COUNTS
C     INTO SEPARATE FILES FOR EACH FISCAL YEAR.
C
      INTEGER YEAR,OF,GRADE,LOS,INV,L1,L2,L3,L4,
     *L5,L6,L7,L8
      DO 100 I=1,16093
      READ (15,20,END=200) YEAR,OF,GRADE,LOS,INV,
     *L1,L2,L3,L4,L5,L6,L7,L8
   20 FORMAT (4I2,9I5)
      IF (YEAR.EQ.77) WRITE (30,8) YEAR,OF,GRADE,
     *LOS,INV,L1,L2,L3,L4,L5,L6,L7,L8
      IF (YEAR.EQ.78) WRITE (30,9) YEAR,OF,GRADE,
     *LOS,INV,L1,L2,L3,L4,L5,L6,L7,L8
      IF (YEAR.EQ.79) WRITE (30,10) YEAR,OF,GRADE
     *,LOS,INV,L1,L2,L3,L4,L5,L6,L7,L8
      IF (YEAR.EQ.80) WRITE (30,11) YEAR,OF,GRADE
     *,LOS,INV,L1,L2,L3,L4,L5,L6,L7,L8
      IF (YEAR.EQ.81) WRITE (30,12) YEAR,OF,GRADE
     *,LOS,INV,L1,L2,L3,L4,L5,L6,L7,L8
      IF (YEAR.EQ.82) WRITE (30,13) YEAR,OF,GRADE
     *,LOS,INV,L1,L2,L3,L4,L5,L6,L7,L8
      IF (YEAR.EQ.83) WRITE (30,14) YEAR,OF,GRADE
     *,LOS,INV,L1,L2,L3,L4,L5,L6,L7,L8
   30 FORMAT (4I2,9I5)
  100 CONTINUE
  200 STOP
      END
$ENTRY
```

Figure D.3 WATFIV Program SORT

Ensure the read and write files are properly defined for each year.

Use the WATFIV program LOSS in Figure D.5 to create an array of loss indices for each fiscal year, similar to the inventory indices in step 5 above. Note that the losses are aggregated, in that the loss data on the COUNXX DATA file is broken into 8 different loss categories (see Reference 1). Such a breakdown is not used in the present work. See Tucker [Ref. 1] for programs and procedures if loss type data is desired. Again, the data can be read into files named LOSSXX DATA, where XX is the fiscal year. The arrays LOSSXX will be significantly smaller than the INVXX arrays.

Finally, use the APL system exec CMSIO to move the data files INVXX and LOSSXX into an APL workspace. The APL arrays

```
$JOB
      INTEGER GRADE,OF,YEAR,LOS,INDEX,IN,TOT,IYR
C
C     THE DO LOOPS CORRESPOND TO THE NUMBER OF
C        RECORDS FOR YEARS 1977 THRU 1983.
C
      DO 100 I=1,2203
C     DO 100 I=1,2231
C     DO 100 I=1,2337
C     DO 100 I=1,2351
C     DO 100 I=1,2317
C     DO 100 I=1,2324
C     DO 100 I=1,2330
      READ (10,20,END=200) YEAR,OF,GRADE,LOS,INV
     *,L1,L2,L3,L4,L5,L6,L7,L8
   20 FORMAT (4I2,9I5)
      IYR = YEAR-76
      TOT = 100000000*IYR
      IF (INV.GT.0) CALL SUM (IN,TOT,OF,GRADE,LOS
     *,INV)
  100 CONTINUE
  200 STOP
      END
C
C     THIS SUBROUTINE CREATES THE INDEX ARRAY
C         FOR AN INVENTORY DATA ELEMENT.
C
      SUBROUTINE SUM (INDEX,I,J,K,L,NUM)
      INTEGER INDEX,I,J,K,L,NUM
      INDEX = I+J*1000000+K*100000+L*1000+NUM
      WRITE (11,300) INDEX
  300 FORMAT (I9)
      RETURN
      END
$ENTRY
```

Figure D.4 WATFIV Program INV

should be character, vice numeric, arrays, and CMSIO allows this choice. The APL functions INVMATX and MATRIX discussed below assume the APL character arrays are words of ten characters, the first nine characters being the data index, and the tenth character a blank.

3. CREATING THE INVENTORY AND LOSS ARRAYS

Using the INVXX arrays created above, and the APL functions GETINV in Figure D.6 and INVMATX in Figure D.7, create the arrays IXX. Note that GETINV calls INVMATX, and INVMATX uses the INVXX arrays. APL workspace size limitations may be a problem due to the large amount of data, and it may be

```
$JOB
      INTEGER GRADE,OF,YEAR,LOS,IYR,LOSS
C
C     THE DO LOOPS CORRESPOND TO THE NUMBER OF
C        RECORDS FOR YEARS 1977 THRU 1983.
C
      DO 100 I=1,2203
C     DO 100 I=1,2231
C     DO 100 I=1,2337
C     DO 100 I=1,2351
C     DO 100 I=1,2317
C     DO 100 I=1,2324
C     DO 100 I=1,2330
      READ (10,20,END=200) YEAR,OF,GRADE,LOS,INV,
     *L1,L2,L3,L4,L5,L6,L7,L8
   20 FORMAT (4I2,9I5)
      IYR = YEAR-76
      LOSS = 100000000*IYR
      IF (L1.GT.0) CALL SUM (IN,LOSS,OF,GRADE,LOS
     *,L1)
      IF (L2.GT.0) CALL SUM (IN,LOSS,OF,GRADE,LOS
     *,L2)
      IF (L3.GT.0) CALL SUM (IN,LOSS,OF,GRADE,LOS
     *,L3)
      IF (L4.GT.0) CALL SUM (IN,LOSS,OF,GRADE,LOS
     *,L4)
      IF (L5.GT.0) CALL SUM (IN,LOSS,OF,GRADE,LOS
     *,L5)
      IF (L6.GT.0) CALL SUM (IN,LOSS,OF,GRADE,LOS
     *,L6)
      IF (L7.GT.0) CALL SUM (IN,LOSS,OF,GRADE,LOS
     *,L7)
      IF (L8.GT.0) CALL SUM (IN,LOSS,OF,GRADE,LOS
     *,L8)
  100 CONTINUE
  200 STOP
      END
C
C     THIS SUBROUTINE CREATES THE INDEX ARRAY
C           FOR A LOSS DATA ELEMENT.
C
      SUBROUTINE SUM (INDEX,I,J,K,L,NUM)
      INTEGER INDEX,I,J,K,L,NUM
      INDEX = I+J*1000000+K*100000+L*1000+NUM
      WRITE (11,300) INDEX
  300 FORMAT (I9)
      RETURN
      END
$ENTRY
```

Figure D.5 WATFIV Program LOSS

necessary to create one or two arrays at a time, and then copy them to another workspace.

The LXX arrays are created in a manner similar to the above, using the APL functions GETLOSS in Figure D.8, and MATRIX in Figure D.9. MATRIX uses the loss arrays LOSSXX.

```
      ∇ GETINV
      ⍝ THIS FUNCTION CALLS THE FUNCTION INVMATX
      ⍝ FOR EACH FISCAL YEAR. IXX IS THE INVENTORY
      ⍝ ARRAY FOR FISCAL YEAR XX BY OF/LOS/GRADE.
        I77←INVMATX INV77
        I78←INVMATX INV78
        I79←INVMATX INV79
        I80←INVMATX INV80
        I81←INVMATX INV81
        I82←INVMATX INV82
        I83←INVMATX INV83
        ' SHAPE OF I77 IS '
        ⎕←ρI77
      ∇
```

Figure D.6 APL Function GETINV

```
      ∇ Z←INVMATX X;A;B;C;D;E;F;I;J
      ⍝ CREATES THE INVENTORY ARRAYS FOR THE FISCAL
      ⍝ YEARS USING THE ARRAYS OF INDEXES INVXX.
      ⍝ INVXX MUST BE A CHARACTER VECTOR OF 9 DATA
      ⍝ ENTRIES FOLLOWED BY 1 BLANK FOR EACH LOOP.
        Z←(40 31 10)ρ0
        I←ρX
        J←(I+1)÷10
      LOOP:→(J≡0)/OUT
      ⍝ A←⍎(1↑X)
        B←1+(⍎(2↑X←(1↓X)))
        C←1+(⍎(1↑X←(2↓X)))
        D←1+(⍎(2↑X←(1↓X)))
        E←⍎(3↑X←(2↓X))
        Z[B;D;C]←E
        X←(4↓X)
        J←J-1
        →LOOP
      OUT:'FINISHED -- SHAPE OF MATRIX IS '
        ρZ
      ∇
```

Figure D.7 APL Function INVMATX

To create the aggregate (e.g., aviation, combat support, ground combat) inventory arrays, use the APL functions GETAV in Figure D.10, GETCS in Figure D.11, GETGC in Figure D.12, and GETOF in Figure D.13. GETOF is called by the other three functions. Each calling function creates on aggregate array,

```
    ∇ GETLOSS
  ⍝ THIS FUNCTION CALLS MATRIX FOR EACH FISCAL
  ⍝ YEAR. LXX IS THE LOSS ARRAY FOR FISCAL YEAR
  ⍝ XX BY OF/LOS/GRADE.
   L77←MATRIX LOSS77
   L78←MATRIX LOSS78
   L79←MATRIX LOSS79
   L80←MATRIX LOSS80
  ⍝ L81←MATRIX LOSS81
  ⍝ L82←MATRIX LOSS82
  ⍝ L83←MATRIX LOSS83
    ∇
```

Figure D.8 APL Function GETLOSS

```
    ∇ Z←MATRIX X;A;B;C;D;E;F;I;J
  ⍝ THIS FUNCTION CREATES THE LOSS ARRAY FOR
  ⍝ THE FISCAL YEARS USING THE ARRAY OF LOSS
  ⍝ INDICES LOSSXX. IT IS CALLED BY GETLOSS.
  ⍝ LOSSXX MUST BE A CHARACTER VECTOR WITH 9
  ⍝ DATA ENTRIES FOLLOWED BY 1 BLANK FOR EACH
  ⍝ LOOP.
   Z←(40 31 10)⍴0
   I←⍴X
   J←(I+1)÷10
  LOOP:→(J≡0)/OUT
   A←⍎(1↑X)
   B←1+(⍎(2↑X←(1↓X)))
   C←1+(⍎(1↑X←(2↓X)))
   D←1+(⍎(2↑X←(1↓X)))
   E←⍎(1↑X←(2↓X))
   F←⍎(2↑X←(1↓X))
   Z[B;D;C]←Z[B,D;C]+F
   X←(3↓X)
   J←J-1
   →LOOP
  OUT:'FINISHED -- SHAPE OF MATRIX IS'
   ⍴Z
    ∇
```

Figure D.9 APL Function MATRIX

e.g., GETGC creates the ground combat inventory array named GC. Again, workspace size limitations may be a problem, and the functions can be altered to create part of an array at a time. This permits the operator to reduce the number of IXX arrays present in the workspace. Note that the arrays are a

'central inventory' array, taking the maximum of the yearly loss and the average of the year's inventory.

```
      ∇ GETAV
[1]   ⍝ THIS CREATES THE INVENTORY MATRIX FOR
-2-   ⍝ THE AVIATION GROUP. CALLS GETOF.
-3-   ⍝ OCCUPATION GROUP. CALLS GETOF.
[4]   ⍝ USE LINES 6-7 FOR ESTIMATION INVENTORY
[5]   ⍝ MATRIX.
[6]    AV←(4 31 10)⍴0
-7-    AV-GETOF 38
[8]   ⍝ USE LINES 9-11 FOR VALIDATION INVENTORY
[9]   ⍝ MATRIX.
[10]   VAV←(4 31 10)⍴0
-11-   VAV-GETOF 38
      ∇
```

Figure D.10 APL Function GETAV

```
      ∇ GETCS
[1]   ⍝ THIS CREATES THE INVENTORY MATRIX FOR THE
-2-   ⍝ COMBAT SUPPORT GROUP. CALLS GETOF.
[3]   ⍝ USE LINES 5-12 FOR ESTIMATION INVEN-
[4]   ⍝ TORY MATRIX
[5]    CS←(3 4 31 10)⍴0
[6]   ⍝ THIS CREATES THE OF ENGINEERS
-7-    CS-1;;;--GETOF 7
[8]   ⍝ THIS CREATES THE OF OPERATIONAL COMMUN-
[9]   ⍝ ICATIONS
-10-   CS-2;;;--GETOF 13
[11]  ⍝ THIS CREATES THE OF MOTOR TRANSPORT
-12-   CS-3;;;--GETOF 20
[13]  ⍝ USE LINES 15-22 FOR VALIDATION INVEN-
[14]  ⍝ TORY MATRIX
[15]   VCS←(3 4 31 10)⍴0
[16]  ⍝ THIS CREATES THE OF ENGINEERS
-17-   VCS-1;;;--GETOF 7
[18]  ⍝ THIS CREATES THE OF OPERATIONAL COMMUN-
[19]  ⍝ ICATIONS
-20-   VCS-2;;;--GETOF 13
[21]  ⍝ THIS CREATES THE OF MOTOR TRANSPORT
-22-   VCS-3;;;--GETOF 20
      ∇
```

Figure D.11 APL Function GETCS

The aggregate loss arrays (e.g., AVL, CSL, GCL, etc.) are now created using the APL functions GCLOSS in Figure

```
      ∇ GETGC
[1]   ⍝ THIS CREATES THE INVENTORY MATRIX FOR THE
[2]   ⍝ GROUND COMBAT OCCUPATION GROUP. CALLS
-3-   ⍝ GETOF.
[4]   ⍝ USE LINES 6-12 FOR ESTIMATION INVEN-
[5]   ⍝ TORY MATRIX
[6]     GC←(3 4 31 10)⍴0
[7]   ⍝  THIS CREATES THE OF INFANTRY
-8-     GC-1;;;--GETOF 3
[9]   ⍝  THIS CREATES THE OF ARTILLERY
-10-    GC-2;;;--GETOF 5
[11]  ⍝  THIS CREATES THE OF TANKS AND AMPHIB
-12-    GC-3;;;--GETOF 10
[13]  ⍝ USE LINES 15-21 FOR VALIDATION INVEN-
[14]  ⍝ TORY MATRICES
[15]    VGC←(3 3 31 10)⍴0
[16]  ⍝  THIS CREATES THE OF INFANTRY
-17-    VGC-1;;;--GETOF 3
[18]  ⍝  THIS CREATES THE OF ARTILLERY
-19-    VGC-2;;;--GETOF 5
[20]  ⍝  THIS CREATES THE OF TANKS AND AMPHIB
-21-    VGC-3;;;--GETOF 10
      ∇
```

Figure D.12 APL Function GETGC

```
      - Z-GETOF X;J;Y
[1]   ⍝ CREATES THE CENTRAL INVENTORY FOR THE YEAR
[2]   ⍝ DESIRED USING THE ARRAYS IXX.
[3]     J←X+1
[4]   ⍝ USE LINES 5-9 FOR THE ESTIMATION MATRIX.
[5]     Z←(4 31 10)⍴0
[6]     Z[1;;]←((Y←(I77[J;;]+I78[J;;])÷2)⌈(L77[J;;]))
[7]     Z[2;;]←((Y←(I78[J;;]+I79[J;;])÷2)⌈(L78[J;;]))
[8]     Z[3;;]←((Y←(I79[J;;]+I80[J;;])÷2)⌈(L79[J;;]))
[9]     Z[4;;]←((Y←(I80[J;;]+I81[J;;])÷2)⌈(L80[J;;]))
[10]  ⍝ USE LINES 10-14 FOR THE VALIDATION MATRIX.
[11]  ⍝ Z←(3 31 10)⍴0
[12]  ⍝ Z[1;;]←((Y←(I81[J;;]+I82[J;;])÷2)⌈(L81[J;;]))
[13]  ⍝ Z[2;;]←((Y←(I82[J;;]+I83[J;;])÷2)⌈(L82[J;;]))
[14]  ⍝ Z[3;;]←I83[J;;]⌈L83[J;;]
      ∇
```

Figure D.13 APL Function GETOF

D.14, AVLOSS in Figure D.15, and CSLOSS in Figure D.16. These functions call the arrays LXX directly. Again, workspace size may require creative changes to the functions.

```
      ∇ AVLOSS
[1]   ⍝ THIS CREATES THE LOSS MATRIX FOR
[2]   ⍝ AVIATION.
[3]   ⍝ USE LINES 4-8 FOR ESTIMATION MATRIX.
[4]    AVL←(4 31 10)⍴0
[5]    AVL[1;;]←L77[39;;]
[6]    AVL[2;;]←L78[39;;]
[7]    AVL[3;;]←L79[39;;]
[8]    AVL[4;;]←L80[39;;]
[9]   ⍝ USE LINES 10-13 FOR VALIDATION MATRIX.
[10]   VAVL←(3 31 10)⍴0
[11]   VAVL[1;;]←L81[39;;]
[12]   VAVL[2;;]←L82[39;;]
[13]   VAVL[3;;]←L83[39;;]
      ∇
```

Figure D.14   APL Function AVLOSS

```
      ∇ CSLOSS
[1]   ⍝ THIS CREATES THE LOSS MATRIX FOR COMBAT
[2]   ⍝ SUPPORT.
[3]   ⍝ USE LINES 4-16 FOR ESTIMATION MATRIX.
[4]    CSL←(3 4 31 10)⍴0
[5]    CSL[1;1;;]←L77[8;;]
[6]    CSL[1;2;;]←L78[8;;]
[7]    CSL[1;3;;]←L79[8;;]
[8]    CSL[1;4;;]←L80[8;;]
[9]    CSL[2;1;;]←L77[14;;]
[10]   CSL[2;2;;]←L78[14;;]
[11]   CSL[2;3;;]←L79[14;;]
[12]   CSL[2;4;;]←L80[14;;]
[13]   CSL[3;1;;]←L77[21;;]
[14]   CSL[3;2;;]←L78[21;;]
[15]   CSL[3;3;;]←L79[21;;]
[16]   CSL[3;4;;]←L80[21;;]
[17]  ⍝ USE LINES 18-27 FOR VALIDATION MATRIX.
[18]   VCSL←(3 3 31 10)⍴0
[19]   VCSL[1;1;;]←L81[8;;]
[20]   VCSL[1;2;;]←L82[8;;]
[21]   VCSL[1;3;;]←L83[8;;]
[22]   VCSL[2;1;;]←L81[14;;]
[23]   VCSL[2;2;;]←L82[14;;]
[24]   VCSL[2;3;;]←L83[14;;]
[25]   VCSL[3;1;;]←L81[21;;]
[26]   VCSL[3;2;;]←L82[21;;]
[27]   VCSL[3;3;;]←L83[21;;]
      ∇
```

Figure D.15   APL Function CSLOSS

```
      ∇ GCLOSS
[1]   ⍝ THIS CREATES THE LOSS MATRIX FOR GROUND
[2]   ⍝ COMBAT.
[3]   ⍝ USE LINES 4-16 FOR ESTIMATION MATRIX.
[4]    GCL←(3 4 31 10)ρ0
[5]    GCL[1;1;;]←L77[4;;]
[6]    GCL[1;2;;]←L78[4;;]
[7]    GCL[1;3;;]←L79[4;;]
[8]    GCL[1;4;;]←L80[4;;]
[9]    GCL[2;1;;]←L77[6;;]
[10]   GCL[2;2;;]←L78[6;;]
[11]   GCL[2;3;;]←L79[6;;]
[12]   GCL[2;4;;]←L80[6;;]
[13]   GCL[3;1;;]←L77[11;;]
[14]   GCL[3;2;;]←L78[11;;]
[15]   GCL[3;3;;]←L79[11;;]
[16]   GCL[3;4;;]←L80[11;;]
[17]  ⍝ USE LINES 18-27 FOR VALIDATION MATRIX.
[18]   VGCL←(3 3 31 10)ρ0
[19]   VGCL[1;1;;]←L81[4;;]
[20]   VGCL[1;2;;]←L82[4;;]
[21]   VGCL[1;3;;]←L83[4;;]
[22]   VGCL[2;1;;]←L81[6;;]
[23]   VGCL[2;2;;]←L82[6;;]
[24]   VGCL[2;3;;]←L83[6;;]
[25]   VGCL[3;1;;]←L81[11;;]
[26]   VGCL[3;2;;]←L82[11;;]
[27]   VGCL[3;3;;]←L83[11;;]
      ∇
```

Figure D.16 APL Function GCLOSS

Use the APL functions GETNA in Figure D.17, GETNC in Figure D.18, and GETNG in Figure D.19 to create the desired grade specific inventory arrays, e.g., NA5, which is the central inventory array for aviation First Lieutenants (grade code 5). See Table 17 for grade codes. These functions call the aggregate inventory arrays, i.e., AV, CS, and GC.

The APL functions GETYA in Figure D.20, GETYC in Figure D.21, and GETYG in Figure D.22 can now be used to create the desired grade specific loss arrays, e.g., YA5, which is the loss array for aviation First Lieutenants. These functions use the aggregate loss arrays, i.e., AVL, CSL, and GCL.

Use the APL functions GETANA in Figure D.23, GETANC in Figure D.24, and GETANG in Figure D.25 to create the arrays

```
      ∇ GETNA
[1]   ⍝ THIS CREATES THE GRADE SPECIFIC INVEN-
[2]   ⍝ TORY FOR AVIATION.
[3]   ⍝ USE LINES 4-13 FOR ESTIMATION MATRICES.
[4]    NA0←(3 1 2)⍉(1 4 31)ρAV[;;1]
[5]    NA1←(3 1 2)⍉(1 4 31)ρAV[;;2]
[6]    NA2←(3 1 2)⍉(1 4 31)ρAV[;;3]
[7]    NA3←(3 1 2)⍉(1 4 31)ρAV[;;4]
[8]    NA4←(3 1 2)⍉(1 4 31)ρAV[;;5]
[9]    NA5←(3 1 2)⍉(1 4 31)ρAV[;;6]
[10]   NA6←(3 1 2)⍉(1 4 31)ρAV[;;7]
[11]   NA7←(3 1 2)⍉(1 4 31)ρAV[;;8]
[12]   NA8←(3 1 2)⍉(1 4 31)ρAV[;;9]
[13]   NA9←(3 1 2)⍉(1 4 31)ρAV[;;10]
[14]  ⍝ USE LINES 15-24 FOR VALIDATION MATRICES.
[15]   VNA0←(3 1 2)⍉(1 3 31)ρVAV[;;1]
[16]   VNA1←(3 1 2)⍉(1 3 31)ρVAV[;;2]
[17]   VNA2←(3 1 2)⍉(1 3 31)ρVAV[;;3]
[18]   VNA3←(3 1 2)⍉(1 3 31)ρVAV[;;4]
[19]   VNA4←(3 1 2)⍉(1 3 31)ρVAV[;;5]
[20]   VNA5←(3 1 2)⍉(1 3 31)ρVAV[;;6]
[21]   VNA6←(3 1 2)⍉(1 3 31)ρVAV[;;7]
[22]   VNA7←(3 1 2)⍉(1 3 31)ρVAV[;;8]
[23]   VNA8←(3 1 2)⍉(1 3 31)ρVAV[;;9]
[24]   VNA9←(3 1 2)⍉(1 3 31)ρVAV[;;10]
       ∇
```

Figure D.17 APL Function GETNA

of grade specific average inventory. For example, ANA5 is the average inventory array for aviation First Lieutenants. The above three functions call GETAAV in Figure D.26, GETACS in Figure D.27, and GETAGC in Figure D.28, respectively. Also, this second group of three functions all call GETAOF in Figure D.29, which in turn uses the arrays IXX.

The procedures have now produced central inventory and loss arrays for the estimation and validation years, and the average inventory arrays for the estimation years. All of these arrays are needed to calculate attrition rates and figures of merit. Note there is no requirement to create arrays of the average losses.

```
      ∇ GETNC
[1]   ⍝ THIS CREATES THE INVENTORY FOR COMBAT
[2]   ⍝ SUPPORT.
[3]   ⍝ USE LINES 4-13 FOR ESTIMATION MATRIX.
[4]    NC0←(3 1 2)⍉CS[;;;1]
[5]    NC1←(3 1 2)⍉CS[;;;2]
[6]    NC2←(3 1 2)⍉CS[;;;3]
[7]    NC3←(3 1 2)⍉CS[;;;4]
[8]    NC4←(3 1 2)⍉CS[;;;5]
[9]    NC5←(3 1 2)⍉CS[;;;6]
[10]   NC6←(3 1 2)⍉CS[;;;7]
[11]   NC7←(3 1 2)⍉CS[;;;8]
[12]   NC8←(3 1 2)⍉CS[;;;9]
[13]   NC9←(3 1 2)⍉CS[;;;10]
[14]  ⍝ USE LINES 15-24 FOR VALIDATION MATRIX.
[15]   VNC0←(3 1 2)⍉VCS[;;;1]
[16]   VNC1←(3 1 2)⍉VCS[;;;2]
[17]   VNC2←(3 1 2)⍉VCS[;;;3]
[18]   VNC3←(3 1 2)⍉VCS[;;;4]
[19]   VNC4←(3 1 2)⍉VCS[;;;5]
[20]   VNC5←(3 1 2)⍉VCS[;;;6]
[21]   VNC6←(3 1 2)⍉VCS[;;;7]
[22]   VNC7←(3 1 2)⍉VCS[;;;8]
[23]   VNC8←(3 1 2)⍉VCS[;;;9]
[24]   VNC9←(3 1 2)⍉VCS[;;;10]
      ∇
```

Figure D.18 APL Function GETNC

4. ATTRITION RATE AND FOM CALCULATIONS

The remainder of the procedures assumes certain global APL variables are defined in the workspace:

(1) N = the estimation period central inventory array, e.g., NA5,

(2) Y = the estimation period loss array, e.g., YA5,

(3) VN = the validation period central inventory array,e.g., VNA5,

(4) VY = the validation period loss array, e.g., VYA5,

(5) AN = the estimation period average inventory array, e.g., ANA5, and

(6) G = the James-Stein forced shrinkage rate.

All functions are invoked by entering the function name. Since the above global variables and the global output variables from functions discussed below are used to calculate attrition rates and FOM's, care must be taken to ensure variable values are not changed inadvertently.

```
      ∇ GETNG
[1]   ⍝ THIS CREATES THE GRADE SPECIFIC INVEN-
[2]   ⍝ TORY FOR GROUND COMBAT.
[3]   ⍝ USE LINES 4-13 FOR ESTIMATION MATRIX.
[4]    NG0←(3 1 2)⍉GC[;;;1]
[5]    NG1←(3 1 2)⍉GC[;;;2]
[6]    NG2←(3 1 2)⍉GC[;;;3]
[7]    NG3←(3 1 2)⍉GC[;;;4]
[8]    NG4←(3 1 2)⍉GC[;;;5]
[9]    NG5←(3 1 2)⍉GC[;;;6]
[10]   NG6←(3 1 2)⍉GC[;;;7]
[11]   NG7←(3 1 2)⍉GC[;;;8]
[12]   NG8←(3 1 2)⍉GC[;;;9]
[13]   NG9←(3 1 2)⍉GC[;;;10]
[14]  ⍝ USE LINES 15-24 FOR VALIDATION MATRIX.
[15]   VNG0←(3 1 2)⍉VGC[;;;1]
[16]   VNG1←(3 1 2)⍉VGC[;;;2]
[17]   VNG2←(3 1 2)⍉VGC[;;;3]
[18]   VNG3←(3 1 2)⍉VGC[;;;4]
[19]   VNG4←(3 1 2)⍉VGC[;;;5]
[20]   VNG5←(3 1 2)⍉VGC[;;;6]
[21]   VNG6←(3 1 2)⍉VGC[;;;7]
[22]   VNG7←(3 1 2)⍉VGC[;;;8]
[23]   VNG8←(3 1 2)⍉VGC[;;;9]
[24]   VNG9←(3 1 2)⍉VGC[;;;10]
      ∇
```

Figure D.19 APL Function GETNG

Use the APL function ESTIM in Figure D.30 to calculate the array R of transformed scale attrition rate estimates. This function calls BINPREP in Figure D.31, SUMSQ in Figure D.32, and MLE in Figure D.33. The first page of R is the array of original scale aggregate rates, each entry being equal. The second page is the array of transformed scale aggregate rates. The third page is the cell MLE rates. The fourth page is the TSCA estimates. The fifth page is the James-Stein estimates, and the sixth and subsequent pages (if any) are the limited translation James-Stein rate estimates for different values of d.

The user now has several options. First, the relative savings loss for different values of d can be calculated using the APL function RELS in Figure D.34. Enter ESTIM to select the d values, ensuring the shape of R conforms by adding enough limited translation pages to handle all

```
      ∇ GETYA
[1]    ⍝ THIS CREATES THE GRADE SPECIFIC LOSS
[2]    ⍝ ARRAYS FOR AVIATION.
[3]    ⍝ USE LINES 4-13 FOR ESTIMATION MATRICES
[4]     YA0←(3 1 2)⍉(1 4 31)ρAVL[;;1]
[5]     YA1←(3 1 2)⍉(1 4 31)ρAVL[;;2]
[6]     YA2←(3 1 2)⍉(1 4 31)ρAVL[;;3]
[7]     YA3←(3 1 2)⍉(1 4 31)ρAVL[;;4]
[8]     YA4←(3 1 2)⍉(1 4 31)ρAVL[;;5]
[9]     YA5←(3 1 2)⍉(1 4 31)ρAVL[;;6]
[10]    YA6←(3 1 2)⍉(1 4 31)ρAVL[;;7]
[11]    YA7←(3 1 2)⍉(1 4 31)ρAVL[;;8]
[12]    YA8←(3 1 2)⍉(1 4 31)ρAVL[;;9]
[13]    YA9←(3 1 2)⍉(1 4 31)ρAVL[;;10]
[14]   ⍝ USE LINES 13-24 FOR VALIDATION MATRICES
[15]    VYA0←(3 1 2)⍉(1 3 31)ρVAVL[;;1]
[16]    VYA1←(3 1 2)⍉(1 3 31)ρVAVL[;;2]
[17]    VYA2←(3 1 2)⍉(1 3 31)ρVAVL[;;3]
[18]    VYA3←(3 1 2)⍉(1 3 31)ρVAVL[;;4]
[19]    VYA4←(3 1 2)⍉(1 3 31)ρVAVL[;;5]
[20]    VYA5←(3 1 2)⍉(1 3 31)ρVAVL[;;6]
[21]    VYA6←(3 1 2)⍉(1 3 31)ρVAVL[;;7]
[22]    VYA7←(3 1 2)⍉(1 3 31)ρVAVL[;;8]
[23]    VYA8←(3 1 2)⍉(1 3 31)ρVAVL[;;9]
[24]    VYA9←(3 1 2)⍉(1 3 31)ρVAVL[;;10]
      ∇
```

Figure D.20 APL Function GETYA

selected values of d. Next, enter RELS and ensure the shape of the variable RSL (which is the outputed relative savings loss) conforms to the number of d values. Then run RELS. A graph of d versus RSL allows a choice of the best d.

The second option, if a value of d is available, is to calculate the risks of the various estimation methods (in the transformed scale) represented in the R array. Use ESTIM and the APL function RISKT in Figure D.35 for these calculations. ESTIM calculates the R array using a single d value, and RISKT uses this R to calculate risks. The function can only handle an R array that has one page of limited translation rates. Output is by validation year.

Calculation of the risk in the original scale is the third option. Use the function RISKO in Figure D.36 just as RISKT was used. This function calls BINCONV (Figure D.37).

```
      ∇ GETYC
[1]    ⍝ THIS CREATES THE GRADE SPECIFIC LOSS
[2]    ⍝ ARRAYS FOR COMBAT SUPPORT.
[3]    ⍝ USE LINES 4-13 FOR ESTIMATION MATRICES
[4]     YC0←(3 1 2)⍉CSL[;;;1]
[5]     YC1←(3 1 2)⍉CSL[;;;2]
[6]     YC2←(3 1 2)⍉CSL[;;;3]
[7]     YC3←(3 1 2)⍉CSL[;;;4]
[8]     YC4←(3 1 2)⍉CSL[;;;5]
[9]     YC5←(3 1 2)⍉CSL[;;;6]
[10]    YC6←(3 1 2)⍉CSL[;;;7]
[11]    YC7←(3 1 2)⍉CSL[;;;8]
[12]    YC8←(3 1 2)⍉CSL[;;;9]
[13]    YC9←(3 1 2)⍉CSL[;;;10]
[14]   ⍝ USE LINES 15-24 FOR VALIDATION MATRICES
[15]    VYC0←(3 1 2)⍉VCSL[;;;1]
[16]    VYC1←(3 1 2)⍉VCSL[;;;2]
[17]    VYC2←(3 1 2)⍉VCSL[;;;3]
[18]    VYC3←(3 1 2)⍉VCSL[;;;4]
[19]    VYC4←(3 1 2)⍉VCSL[;;;5]
[20]    VYC5←(3 1 2)⍉VCSL[;;;6]
[21]    VYC6←(3 1 2)⍉VCSL[;;;7]
[22]    VYC7←(3 1 2)⍉VCSL[;;;8]
[23]    VYC8←(3 1 2)⍉VCSL[;;;9]
[24]    VYC9←(3 1 2)⍉VCSL[;;;10]
      ∇
```

Figure D.21 APL Function GETYC

Both RISKT and RISKO require an array SS to be defined globally. SS allows selection of the specific cells to be included in the risk outputed by the two functions. If the global risk is desired, SS should be an array of all ones. If certain cells are to be investigated, e.g., cells with inventory less than six, set up SS so that each cell with an inventory less than six has an entry of 1 in SS, and all other cells have an entry of zero. SS must have the same shape as the inventory and loss arrays, e.g., NA5 and YA5.

The fourth option is to calculate the original scale attrition rate estimates using the APL function BINCONV in Figure D.38. Use the array R and the central inventory array for the aggregate being investigated.

```
      ∇ GETYG
[1]   ⍝ THIS CREATES THE GRADE SPECIFIC LOSS
[2]   ⍝ ARRAYS FOR GROUND COMBAT COMBAT.
[3]   ⍝ USE LINES 5-14 FOR ESTIMATION MATRICES
[4]    YG0←(3 1 2)⍉GCL[;;;1]
[5]    YG1←(3 1 2)⍉GCL[;;;2]
[6]    YG2←(3 1 2)⍉GCL[;;;3]
[7]    YG3←(3 1 2)⍉GCL[;;;4]
[8]    YG4←(3 1 2)⍉GCL[;;;5]
[9]    YG5←(3 1 2)⍉GCL[;;;6]
[10]   YG6←(3 1 2)⍉GCL[;;;7]
[11]   YG7←(3 1 2)⍉GCL[;;;8]
[12]   YG8←(3 1 2)⍉GCL[;;;9]
[13]   YG9←(3 1 2)⍉GCL[;;;10]
[14]  ⍝ USE LINES 16-25 FOR VALIDATION MATRICES
[15]   VYG0←(3 1 2)⍉VGCL[;;;1]
[16]   VYG1←(3 1 2)⍉VGCL[;;;2]
[17]   VYG2←(3 1 2)⍉VGCL[;;;3]
[18]   VYG3←(3 1 2)⍉VGCL[;;;4]
[19]   VYG4←(3 1 2)⍉VGCL[;;;5]
[20]   VYG5←(3 1 2)⍉VGCL[;;;6]
[21]   VYG6←(3 1 2)⍉VGCL[;;;7]
[22]   VYG7←(3 1 2)⍉VGCL[;;;8]
[23]   VYG8←(3 1 2)⍉VGCL[;;;9]
[24]   VYG9←(3 1 2)⍉VGCL[;;;10]
       ∇
```

Figure D.22 APL Function GETYG

```
      ∇ GETANA;Z
[1]    ⍝ THIS CREATES THE AVERAGE INVENTORY FOR
[2]    ⍝ AVIATION. CALLS GETAAV.
[3]    ⍝ USE LINES 4-14 FOR ESTIMATION ARRAYS
[4]     Z←GETAAV
[5]     ANA0←(3 1 2)⍉(1 4 31)⍴Z[;;1]
[6]     ANA1←(3 1 2)⍉(1 4 31)⍴Z[;;2]
[7]     ANA2←(3 1 2)⍉(1 4 31)⍴Z[;;3]
[8]     ANA3←(3 1 2)⍉(1 4 31)⍴Z[;;4]
[9]     ANA4←(3 1 2)⍉(1 4 31)⍴Z[;;5]
[10]    ANA5←(3 1 2)⍉(1 4 31)⍴Z[;;6]
[11]    ANA6←(3 1 2)⍉(1 4 31)⍴Z[;;7]
[12]    ANA7←(3 1 2)⍉(1 4 31)⍴Z[;;8]
[13]    ANA8←(3 1 2)⍉(1 4 31)⍴Z[;;9]
[14]    ANA9←(3 1 2)⍉(1 4 31)⍴Z[;;10]
[15]   ⍝ USE LINES 16-26 FOR VALIDATION ARRAYS
[16]    Z←GETAAV
[17]    VANA0←(3 1 2)⍉(1 3 31)⍴Z[;;1]
[18]    VANA1←(3 1 2)⍉(1 3 31)⍴Z[;;2]
[19]    VANA2←(3 1 2)⍉(1 3 31)⍴Z[;;3]
[20]    VANA3←(3 1 2)⍉(1 3 31)⍴Z[;;4]
[21]    VANA4←(3 1 2)⍉(1 3 31)⍴Z[;;5]
[22]    VANA5←(3 1 2)⍉(1 3 31)⍴Z[;;6]
[23]    VANA6←(3 1 2)⍉(1 3 31)⍴Z[;;7]
[24]    VANA7←(3 1 2)⍉(1 3 31)⍴Z[;;8]
[25]    VANA8←(3 1 2)⍉(1 3 31)⍴Z[;;9]
[26]    VANA9←(3 1 2)⍉(1 3 31)⍴Z[;;10]
      ∇
```

Figure D.23   APL Function GETANA

```
      ∇ GETANC
[1]   ⍝ THIS CREATES THE AVERAGE INVENTORY FOR
[2]   ⍝ COMBAT SUPPORT. CALLS GETACS.
[3]   ⍝ USE LINES 4-13 FOR ESTIMATION ARRAYS.
[4]     Z←GETACS
[5]     ANC0←(3 1 2)⍉Z[;;;1]
[6]     ANC1←(3 1 2)⍉Z[;;;2]
[7]     ANC2←(3 1 2)⍉Z[;;;3]
[8]     ANC3←(3 1 2)⍉Z[;;;4]
[9]     ANC4←(3 1 2)⍉Z[;;;5]
[10]    ANC5←(3 1 2)⍉Z[;;;6]
[11]    ANC6←(3 1 2)⍉Z[;;;7]
[12]    ANC7←(3 1 2)⍉Z[;;;8]
[13]    ANC8←(3 1 2)⍉Z[;;;9]
[14]    ANC9←(3 1 2)⍉Z[;;;10]
[15]  ⍝ USE LINES 16-26 FOR VALIDATION ARRAYS.
[16]    Z←GETACS
[17]    VANC0←(3 1 2)⍉Z[;;;1]
[18]    VANC1←(3 1 2)⍉Z[;;;2]
[19]    VANC2←(3 1 2)⍉Z[;;;3]
[20]    VANC3←(3 1 2)⍉Z[;;;4]
[21]    VANC4←(3 1 2)⍉Z[;;;5]
[22]    VANC5←(3 1 2)⍉Z[;;;6]
[23]    VANC6←(3 1 2)⍉Z[;;;7]
[24]    VANC7←(3 1 2)⍉Z[;;;8]
[25]    VANC8←(3 1 2)⍉Z[;;;9]
[26]    VANC9←(3 1 2)⍉Z[;;;10]
      ∇
```

Figure D.24 APL Function GETANC

```
      ∇ GETANG
[1]   ⍝ THIS CREATES THE AVERAGE INVENTORY FOR
[2]   ⍝ GROUND COMBAT.CALLS GETAGC.
[3]   ⍝ USE LINES 4-14 FOR ESTIMATION ARRAYS.
[4]     Z←GETAGC
[5]     ANG0←(3 1 2)⍉Z[;;;1]
[6]     ANG1←(3 1 2)⍉Z[;;;2]
[7]     ANG2←(3 1 2)⍉Z[;;;3]
[8]     ANG3←(3 1 2)⍉Z[;;;4]
[9]     ANG4←(3 1 2)⍉Z[;;;5]
[10]    ANG5←(3 1 2)⍉Z[;;;6]
[11]    ANG6←(3 1 2)⍉Z[;;;7]
[12]    ANG7←(3 1 2)⍉Z[;;;8]
[13]    ANG8←(3 1 2)⍉Z[;;;9]
[14]    ANG9←(3 1 2)⍉Z[;;;10]
[15]  ⍝ USE LINES 16-26 FOR VALIDATION ARRAYS.
[16]    Z←GETAGC
[17]    VANG0←(3 1 2)⍉Z[;;;1]
[18]    VANG1←(3 1 2)⍉Z[;;;2]
[19]    VANG2←(3 1 2)⍉Z[;;;3]
[20]    VANG3←(3 1 2)⍉Z[;;;4]
[21]    VANG4←(3 1 2)⍉Z[;;;5]
[22]    VANG5←(3 1 2)⍉Z[;;;6]
[23]    VANG6←(3 1 2)⍉Z[;;;7]
[24]    VANG7←(3 1 2)⍉Z[;;;8]
[25]    VANG8←(3 1 2)⍉Z[;;;9]
[26]    VANG9←(3 1 2)⍉Z[;;;10]
      ∇
```

Figure D.25 APL Function GETANG

```
      ∇ Z←GETAAV;Z
[1]   ⍝ THIS CREATES THE AVERAGE INVENTORY
-2-   ⍝ MATRIX FOR AVIATION. CALLS GETAOF.
[3]   ⍝ USE LINES 4-5 FOR ESTIMATION INVENTORY MATRIX.
[4]     Z←(4 31 10)ρ0
-5-     Z-GETAOF 38
[6]   ⍝ USE LINES 7-8 FOR VALIDATION INVENTORY MATRIX.
[7]   ⍝ Z←(3 31 10)ρ0
-8-   ⍝ Z-GETAOF 38
      ∇
```

Figure D.26 APL Function GETAAV

```
      ∇ Z←GETACS;Z
[1]   ⍝ THIS CREATES THE AVERAGE INVENTORY
[2]   ⍝ MATRIX FOR COMBAT SUPPORT.
-3-   ⍝ CALLS GETAOF.
[4]   ⍝ USE LINES 5-11 FOR ESTIMATION ARRAYS.
[5]     Z←(3 4 31 10)ρ0
-6-   - THIS GETS THE OF ENGINEERS
-7-     Z-1;;;--GETAOF 7
-8-   - THIS GETS THE OF OPERATIONAL COMM
-9-     Z-2;;;--GETAOF 13
-10-  - THIS GETS THE OF MOTOR TRANSPORT
-11-    Z-3;;;--GETAOF 20
[12]  ⍝ USE LINES 13-19 FOR VALIDATION ARRAYS.
[13]  ⍝ Z←(3 3 31 10)ρ0
-14-  - THIS GETS THE OF ENGINEERS
-15-  ⍝ Z-1;;;--GETAOF 7
-16-  - THIS GETS THE OF OPERATIONAL COMM
-17-  ⍝ Z-2;;;--GETAOF 13
-18-  - THIS GETS THE OF MOTOR TRANSPORT
-19-  ⍝ Z-3;;;--GETAOF 20
      ∇
```

Figure D.27 APL Function GETACS

```
      ∇ Z←GETAGC;Z
[1]   ⍝ THIS CREATES THE AVERAGE INVENTORY
[2]   ⍝ MATRIX FOR GROUND COMBAT GROUP.
-3-   ⍝ CALLS GETAOF.
[4]   ⍝ USE LINES 5-11 FOR ESTIMATION YEARS.
[5]     Z←(3 4 31 10)ρ0
-6-   - THIS GETS THE OF INFANTRY
-7-     Z-1;;;--GETAOF 3
-8-   - THIS GETS THE OF ARTILLERY
-9-     Z-2;;;--GETAOF 5
-10-  - THIS GETS THE OF TANKS AND AMPHIB
-11-    Z-3;;;--GETAOF 10
[12]  ⍝ USE LINES 13-19 FOR VALIDATION YEARS.
[13]  ⍝ Z←(3 3 31 10)ρ0
-14-  - THIS GETS THE OF INFANTRY
-15-  ⍝ Z-1;;;--GETAOF 3
-16-  - THIS GETS THE OF ARTILLERY
-17-  ⍝ Z-2;;;--GETAOF 5
-18-  - THIS GETS THE OF TANKS AND AMPHIB
-19-  ⍝ Z-3;;;--GETAOF 10
      ∇
```

Figure D.28 APL Function GETAGC

```
      ∇ Z←GETAOF X;J;Y;Z
[1]   ⍝ CREATE THE AVERAGE INVENTORY FOR THE
[2]   ⍝ YEARS DESIRED FOR A SPECIFIC OF.
[3]    J←X+1
[4]   ⍝ USE LINES X-XX FOR ESTIMATION ARRAY.
[5]    Z←(4 31 10)⍴0
[6]    Z[1;;]←(I77[J;;]+I78[J;;])÷2
[7]    Z[2;;]←(I78[J;;]+I79[J;;])÷2
[8]    Z[3;;]←(I79[J;;]+I80[J;;])÷2
[9]    Z[4;;]←(I80[J;;]+I81[J;;])÷2
[10]  ⍝ USE LINES XX-XX FOR VALIDATION ARRAY.
[11]  ⍝ Z←(3 31 10)⍴0
[12]  ⍝ Z[1;;]←(I81[J;;]+I82[J;;])÷2
[13]  ⍝ Z[2;;]←(I82[J;;]+I83[J;;])÷2
[14]  ⍝ Z[3;;]←I83[J;;]
      ∇
```

Figure D.29 APL Function GETAOF

```
      ∇ P←ESTIM;I;S;U;C1;C;K;M;ZB;ZBBA;AGO;AGT;D;P
[1]    ⍝ CALCULATES THE AGGREGATE (ORIGINAL),
[2]    ⍝ AGGREGATE (TRANSFORMED), MLE (ORIGINAL),
[3]    ⍝ TSCA (TRANSFORMED), JAMES-STEIN, AND
[4]    ⍝ LIMITED TRANSLATION ESTIMATERS FOR Z AS
[5]    ⍝ SCREENED BY D, THE SCREENING MATRIX FROM
[6]    ⍝ THE ESTIMATION YEARS AVERAGE INVENTORY.
[7]    ⍝ EACH FACE OF R IS A SEPARATE ESTIMATE IN
[8]    ⍝ THE ESTIMATOR ORDER LISTED ABOVE. CHANGE
[9]    ⍝ LINES 12,15,35,36,37 TO CHANGE DEE'S USED.
[10]   ⍝
[11]    Z←Y BINPREP N
[12]    DEE←0
[13]    I←6
[14]    D←(+/ 3 1 2 ⍉AN≠0)≠0
[15]    R←(6,(1↓ρZ))ρ0
[16]    S←SUMSQ Z
[17]    P←(+/,Y×(ρY)ρD)÷+/,N×(ρN)ρD
[18]    AGO←((0.5+(+⌿N÷1↑ρN))*0.5)×¯1○¯1+2×P
[19]    AGT←(+/,ZB)÷+/,D
[20]    ZBBA←(ρZB)ρZBB
[21]    C←0⌈1-SHJ←G
[22]    →(2+1↑⎕LC)×⍳0<G
[23]   ⍝ SHJ IS THE JS SHRINKAGE FACTOR.
[24]    C←0⌈1-SHJ←(K-3)÷(2-K-K×M)×÷/S
[25]    U←|(ZB-ZBBA)×SHJ*0.5
[26]    R[1;;]←(1↓ρR)ρAGO
[27]    R[2;;]←(1↓ρR)ρAGT
[28]    R[3;;]←Y MLE N
[29]    R[4;;]←ZB
[30]    R[5;;]←D×ZBBA+C×ZB-ZBBA
[31]   ⍝ SHL IS ARRAY OF LTJS SHRINKAGE FACTORS.
[32]   LL:C1←0⌈1-SHL←(1⌊DEE÷U)×SHJ
[33]    R[I;;]←D×ZBBA+C1×ZB-ZBBA
[34]   ⍝ USE THE FOLLOWING LINES IF MORE THAN ONE
[35]   ⍝ VALUE OF DEE IS USED.
[36]   ⍝ I←I+1
[37]   ⍝ DEE←DEE+0.2
[38]   ⍝ →LL×⍳(DEE≤1.6)
      ∇
```

Figure D.30 APL Function ESTIM

```
      ∇ Z←Y BINPREP N;P
[1]    ⍝ PREPS THE FREEMAN-TUKEY VERSION OF THE
[2]    ⍝ ARC SIN TRANS FOR BINOMIAL DATA
[3]    ⍝ Y IS LOSSES; N IS INVENTORY
[4]     Z←¯1○¯1+2×Y÷N+1
[5]     Z←0.5×((0.5+N)*0.5)×Z+¯1○¯1+2×(Y+1)÷N+1
      ∇
```

Figure D.31 APL Function BINPREP

```
      ∇ X←SUMSQ Z;SSE
[1]   ⍝ CALCULATES THE SSE AND SSB FOR Z. ALSO
[2]   ⍝ CALCULATES THE MLE (ZB) AND GRAND MEAN
[3]   ⍝ OR AGGREGATE (ZBB), BOTH DIRECTLY FROM
[4]   ⍝ TRANSFORMED DATA.
[5]     K←+/,D
[6]     ZB←D×(+⌿Z)÷M←1↑⍴Z
[7]     ZBB←(+/,ZB)÷K
[8]     X←+/,(Z-(⍴Z)⍴ZBB)*2
[9]     X←(X-SSE),SSE←+/,(Z-(⍴Z)⍴ZB)*2
      ∇
```

Figure D.32   APL Function SUMSQ

```
      ∇ Z←Y MLE N;D;M1
[1]   ⍝ CALCULATES THE MLE IN THE ORIGINAL SCALE
[2]   ⍝ AND TRANSFORMS IT INTO ARCSIN SPACE.
[3]     D←(+/ 3 1 2 ⍉AN≠0)≠0
[4]     M1←((⍴+⌿Y)⍴D)×(+⌿Y)÷+⌿N
[5]     Z←D×((0.5+(+⌿N)÷1↑⍴N)*0.5)×¯1○¯1+2×M1
      ∇
```

Figure D.33   APL Function MLE

```
      ∇RELS[⎕]∇
      ∇ RELS;RM;RJ;RL;I;Z
[1]   ⍝ COMPUTES THE RELATIVE SAVINGS LOSS (RSL).
[2]   ⍝ IN THE TRANSFORMED SCALE. VY IS THE ARRAY
-3-   - OF ACTUAL LOSSES. VN IS THE ARRAY OF
[4]   ⍝ CENTRAL INVENTORY. R IS FROM ESTIM. VN,
[5]   ⍝ VY,R MUST BE IN THE WORKSPACE. CHANGE LINE
[6]   ⍝ 12 FOR DIFFERENT NUMBER OF DEE'S. EACH
[7]   ⍝ FACE OF R IS THE ESTIMATE FOR A DIFFERENT
[8]   ⍝ DEE. CALLS BINPREP.
[9]   ⍝
[10]   I←1
[11]   Z←VY BINPREP VN
[12]   RSL←(9,(1↑⍴Z))⍴0
[13]   RT←+/((1↑⍴Z),(×/(1↓⍴Z)))⍴,(Z-(⍴Z)⍴R[4;;])*2
[14]   RJ←+/((1↑⍴Z),(×/(1↓⍴Z)))⍴,(Z-(⍴Z)⍴R[5;;])*2
[15]  LL:RQ←R[(I+5);;]
[16]   RL←+/((1↑⍴Z),(×/(1↓⍴Z)))⍴,(Z-(⍴Z)⍴RQ)*2
[17]   RSL[I;]←(RL-RJ)÷RT-RJ
[18]   I←I+1
[19]   →LL×⍳((I+6)≤1↑⍴R)
[20]   RSL
      ∇
```

Figure D.34   APL Function RELS

```
∇ RISKT;I;D;RM;RJ;RL;RAO;RAT;RT;Z
⍝ COMPUTES THE SCALED RISK IN THE TRANSFORMED
⍝ SPACE. VY AND VN ARE THE VALIDATION YEAR
⍝ LOSSES AND INVENTORY. VN, VY, R, AND AG
⍝ MUST BE IN THE WORKSPACE.
⍝
 Z←VY BINPREP VN
 I←1
 RAO←RAT←RM←RT←RJ←RL←3⍴0
 D←(+/ 3 1 2 ⍉AN≠0)≠0
LL:RAO[I]←(+/,SS[I;;]×(Z[I;;]-R[1;;])*2)÷+/,D
 RAT[I]←(+/,SS[I;;]×(Z[I;;]-R[2;;])*2)÷+/,D
 RM[I]←(+/,SS[I;;]×(Z[I;;]-R[3;;])*2)÷+/,D
 RT[I]←(+/,SS[I;;]×(Z[I;;]-R[4;;])*2)÷+/,D
 RJ[I]←(+/,SS[I;;]×(Z[I;;]-R[5;;])*2)÷+/,D
 RL[I]←(+/,SS[I;;]×(Z[I;;]-R[6;;])*2)÷+/,D
 I←I+1
 →LL×⍳(I≤3)
 'RISK IN TRANSFORMED SCALE USING THE ESTIM-'
 'ATORS LISTED BELOW.'
 '                    81         82         83'
 'AGG ORIG RISK  ≡ ',(⍕RAO)
 'AGG TRANS RISK ≡ ',(⍕RAT)
 'MLE RISK       ≡ ',(⍕RM)
 'TSCA RISK      ≡ ',(⍕RT)
 'JS RISK        ≡ ',(⍕RJ)
 'LTJS RISK      ≡ ',(⍕RL)
 ' '
∇
```

Figure D.35 APL Function RISKT

```
∇RISKO[⎕]∇
∇ RISKO;D;R1;R2;R3;K;V;B;AR;S;R4;R5;R6;A;RR
⍝ COMPUTES THE RISK IN THE ORIGINAL SPACE.
⍝ VY AND VN ARE THE VALIDATION YEAR LOSSES
⍝ AND INVENTORY. VN,VY,R,N MUST BE IN THE
⍝ WORKSPACE. CALLS BINCONV.
 K←1
 D←∨⌿AN>0
 V←((ρV)ρD)×V←BINCONV
 AR←((ρVY)ρD)×VY÷VN
 S←(V≠0)∧(V≠1)
 R1←R2←R3←R4←R5←R6←(1↑ρVN)ρ0
LM:A←(ρV)ρAR[K;;]
 RR←((ρV)ρD)×S×((ρV)ρVN)×(S×(A-V)*2)÷V×(1-V)
 R1[K]←+/,SS[K;;]×RR[1;;]
 R2[K]←+/,SS[K;;]×RR[2;;]
 KK←+/,D
 KK1←+/,S[3;;]
 R3[K]←+/,SS[K;;]×RR[3;;]×KK÷KK1
 R4[K]←+/,SS[K;;]×RR[4;;]
 R5[K]←+/,SS[K;;]×RR[5;;]
 R6[K]←+/,SS[K;;]×RR[6;;]
 K←K+1
 →LM×ι(K≤1↑ρVN)
 'RISK IN ORIGINAL SCALE USING THE ESTIMATORS'
 'LISTED BELOW.'
 '                     81          82         83'
 'AGG ORIG RISK     ≡ ',(⍕R1)
 'AGG TRANS RISK    ≡ ',(⍕R2)
 'MLE RISK          ≡ ',(⍕R3)
 'TSCA RISK         ≡ ',(⍕R4)
 'JS RISK           ≡ ',(⍕R5)
 'LTJS RISK         ≡ ',(⍕R6)
 ' '
∇
```

Figure D.36 APL Function RISKO

```
      ∇ B←BINCONV;V0;V1;N1;D
[1]    ⍝ INVERTS ARC SIN TRANSFORMATION. R IS THE
[2]    ⍝ ARRAY TO BE INVERTED. N IS THE CENTRAL
[3]    ⍝ INVENTORY FOR THE ESTIMATION YEARS. AN IS
[4]    ⍝ THE AVERAGE INVENTORY FOR THE ESTIMATION
[5]    ⍝ YEARS. R, N, AND AN ARE GLOBAL VARIABLES,
[6]    ⍝ AND MUST BE IN THE WORKSPACE.
[7]      D←∨⌿AN>0
[8]      B←0.5×1+1○R÷(0.5+(N1←(ρR)ρ+⌿N)÷1↑ρN)*0.5
[9]      V0←R<(-○÷2)×(N1+0.5)*0.5
[10]     V1←R>(○÷2)×(N1+0.5)*0.5
[11]     B←((ρR)ρD)×V1+B×V0⍱V1
      ∇
```

Figure D.37 APL Function BINCONV

APPENDIX E

RELATIVE SAVINGS LOSS

1. THEORETICAL IMPLICATIONS OF RELATIVE SAVINGS LOSS

The value of d can be empirically chosen using a concept developed by Efron and Morris [Ref. 5] called the relative savings loss (RSL). The RSL quantifies how well limited translation does versus James-Stein estimation. If

(1) R_T = risk of the TSCA,

(2) R_J = risk of the James-Stein estimator, and

(3) R_{LJ} = risk of the limited translation James-Stein estimator,

then,

$$RSL = (R_{LJ}-R_J)/(R_T-R_J). \tag{E.1}$$

RSL is the proportional increase in global loss if limited translation estimation is used instead of James-Stein estimation.

Efron and Morris [Ref. 5] further state and prove a theorem:

$$RSL = 2[(d^2+1)(1-\Phi(d))-d\varphi(d)] \tag{E.2}$$

where Φ and φ are the standard normal c.d.f. and density function, respectively. By this theorem, RSL is a function of d only. Figure E.1 graphs values of the RSL against d.

2. APPLICATION OF RELATIVE SAVINGS LOSS

Since R_{LJ} is a function of d, a vector of values of R_{LJ} can be calculated for $d \geq 0$. This can yield a vector of values of the RSL. These values can be graphed against the values of d which generated the the RSL vector. Efron and

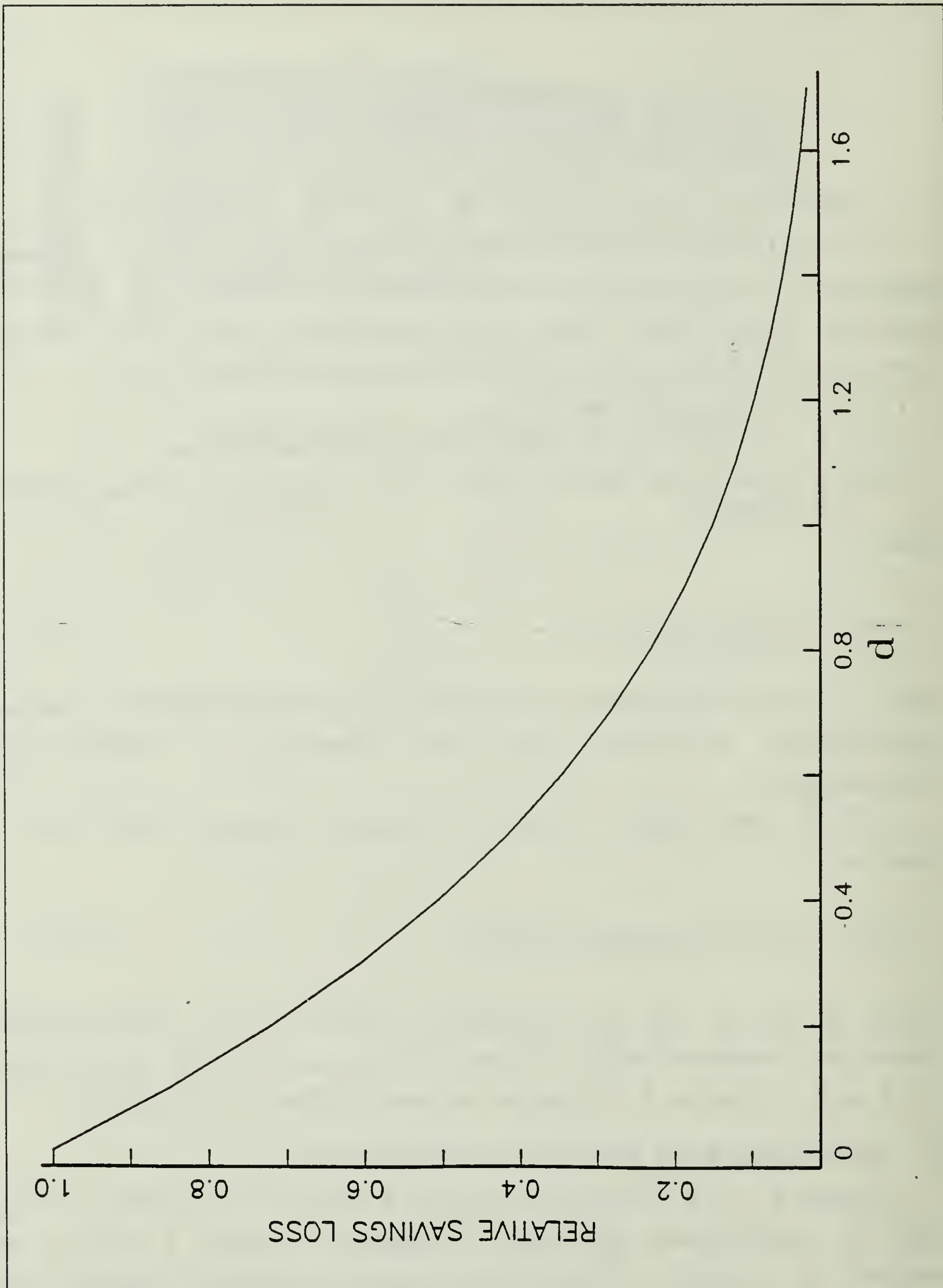

Figure E.1 Theoretical Graph of RSL versus d

Morris [Refs. 5,6] state that, given the normality with common variance assumption, the relationship of the global risks of maximum likelihood, James-Stein, and limited translation will be

$$R_J < R_{LJ} < R_T. \tag{E.3}$$

Tables 4 thru 12 clearly show this is not the case when we identify the validation FOM's as the components of equation E.3.

Figures E.2 thru E.7 are the graphs of calculated RSL versus d for the study cases. The encouraging results are misleading. Tables 4, 5, and 6 display risk values that yield negative quantities in the numerator and denominator of equation E.2. Although the graphs are visually correct, the underlying computations are at variance with theory, and are simply generating offsetting errors. Therefore RSL should not be used to choose d for the present aggregations.

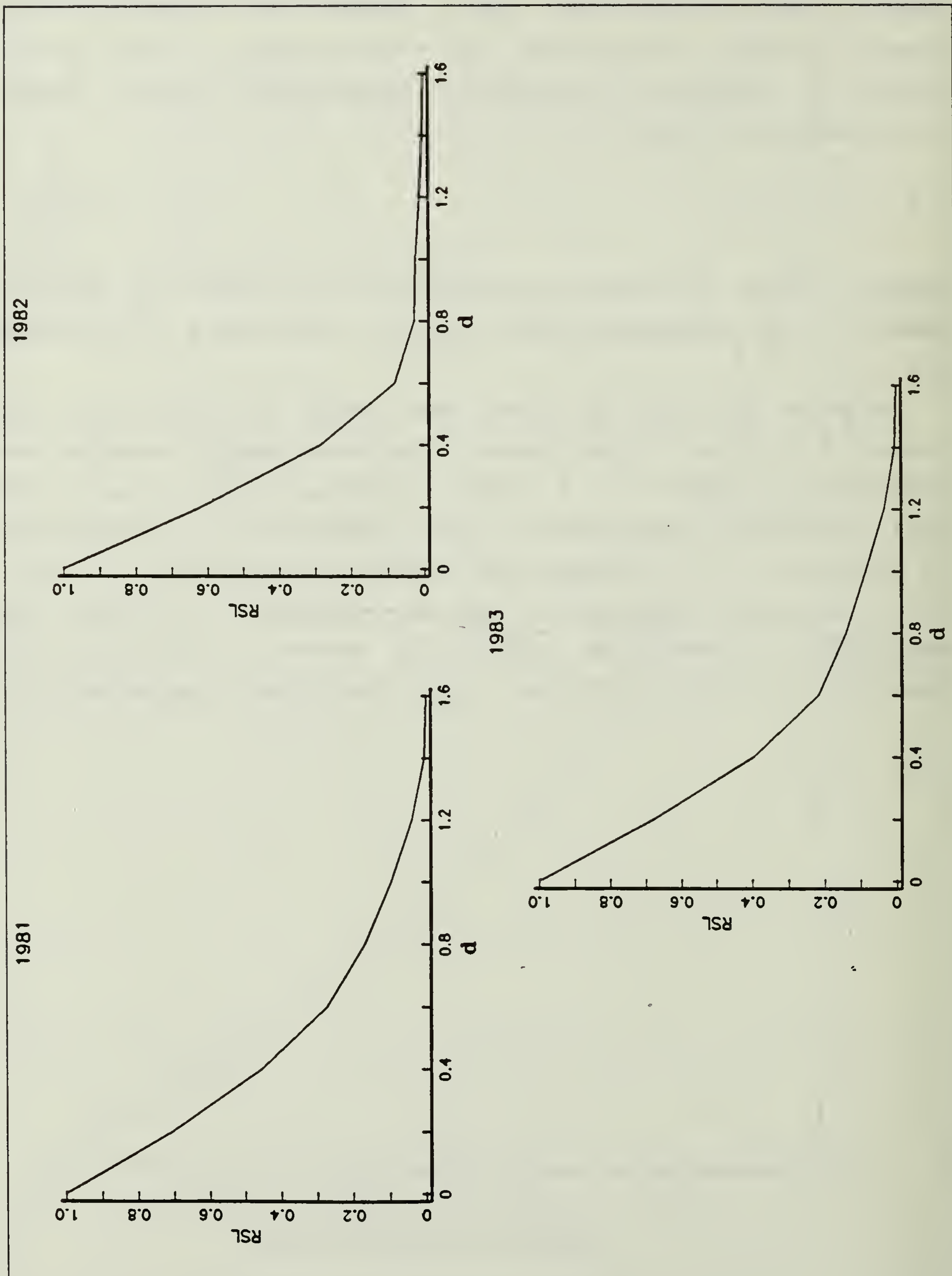

Figure E.2 RSL versus d for Aviation 1st Lieutenants

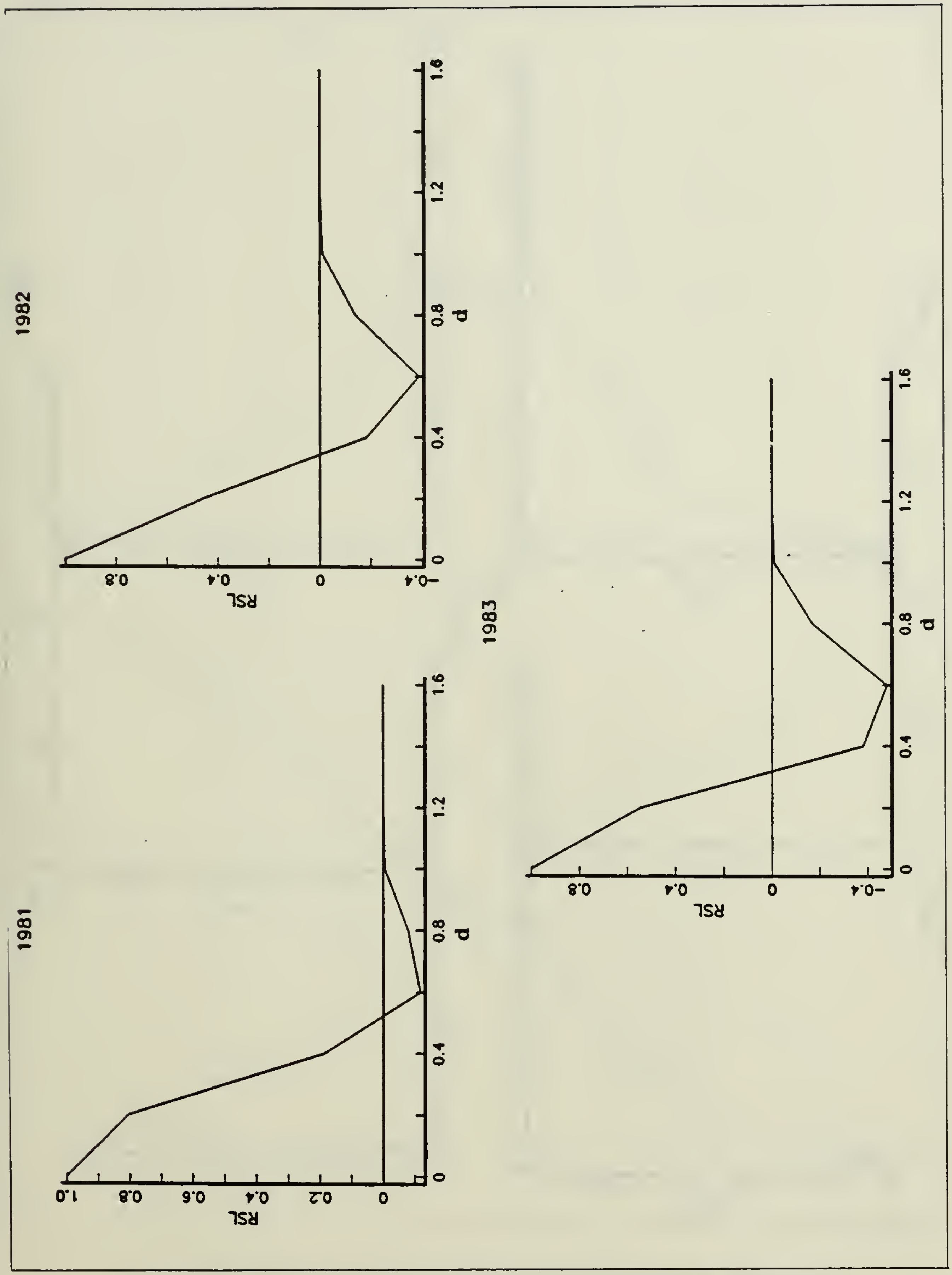

Figure E.3   RSL versus d for Aviation Lieutenant Colonels

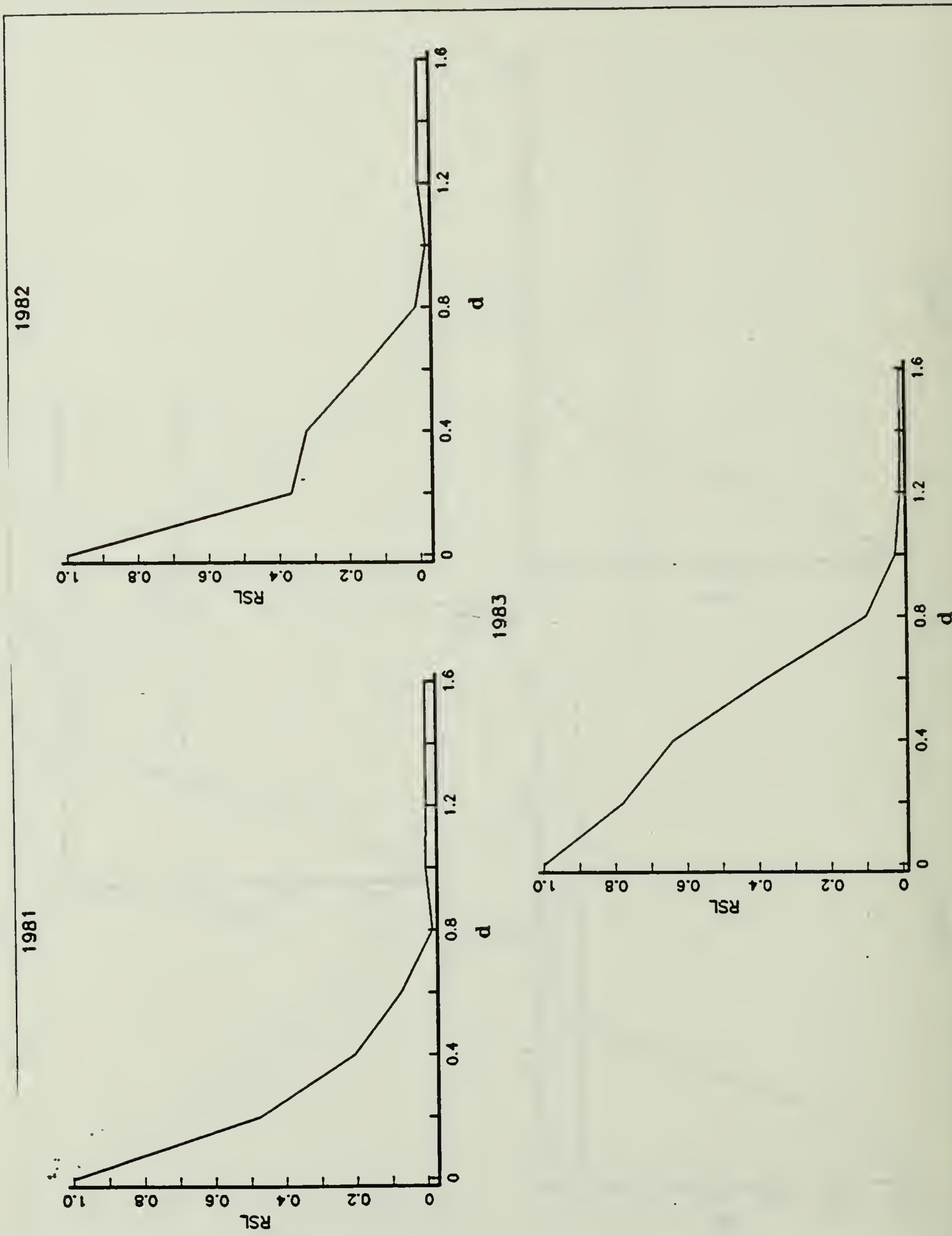

Figure E.4 RSL versus d for Combat Support
1st Lieutenants

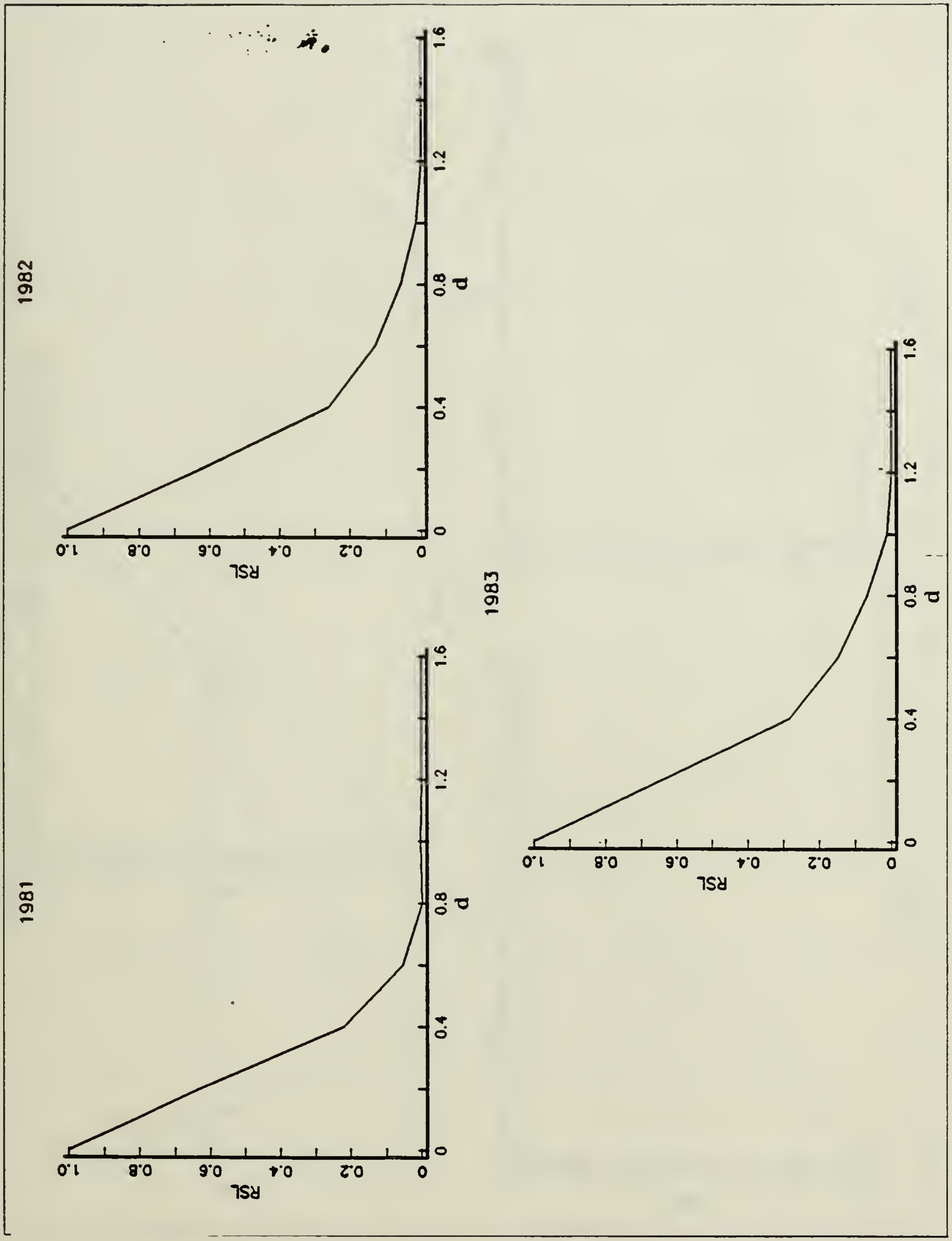

Figure E.5 RSL versus d for Combat Support Lieutenant Colonels

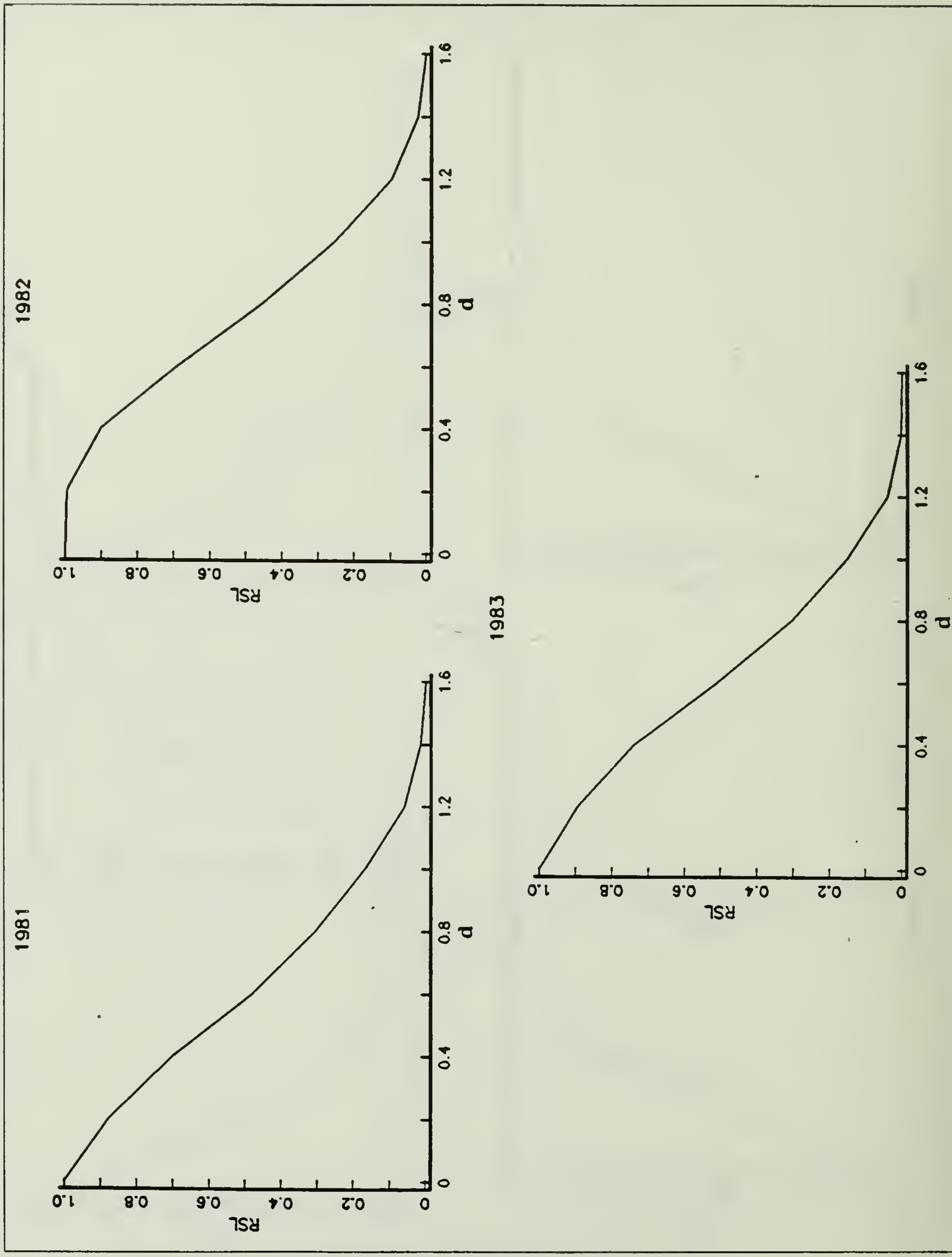

Figure E.6 RSL versus d for Ground Combat 1st Lieutenants

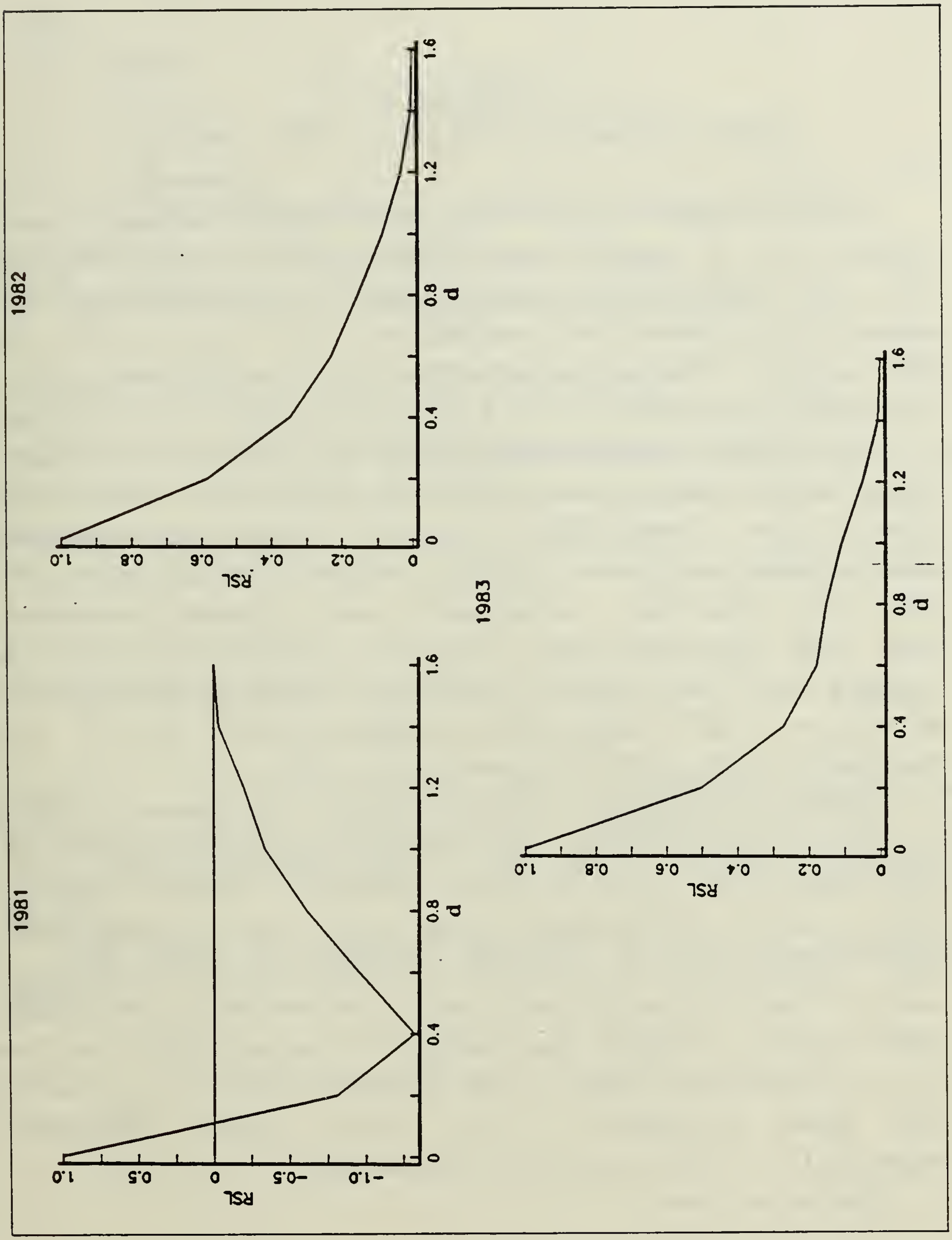

Figure E.7 RSL versus d for Ground Combat Lieutenant Colonels

APPENDIX F

ROBUST PARAMETRIC EMPIRICAL BAYES ANALYSIS

Robust parametric empirical Bayes (RPEB) estimation is a version of a generalized linear model (see McCullagh [Ref. 7]) combined with robust Bayes. It attempts to fit a probabilistic model that assumes the cell value p_i for the i^{th} cell is a random variable that can be described with a probability distribution, e.g., the logistic or the Poisson.

Gaver and O'Muircheartaigh [Ref. 8] developed this technique using the Poisson to model collections of similar objects that independently generate events in accordance with Poisson processes. First, the superpopulation parameters are estimated, using point and interval estimates. Next, the superpopulation parameter estimates are used to compute point and interval estimates of the individual rate parameters. The result is a Bayes estimate of p_i, with limited shrinkage if the inventory is small.

Gaver has suggested, in the spirit of reference 8, using a logistic model for p_i with explanatory variables for OF, LOS, and grade, and an extra binomial variance term to partly explain differences across cells. First, using numerical integration, the maximum likelihood function $L(\mu,\tau)$ is used to find the values of μ and τ that maximize the likelihood function. Second, the expectation of p_i given the loss y_i is calculated. This is the posterior mean of p_i, and is the Bayes estimator of p_i, with limited shrinkage, especially if the inventory n_i is small.

Therefore, let

$$p_i \sim e^m/(1+e^m) = e^q/(1+e^q) \qquad \text{(F.1)}$$

where

$m = x_i\beta+\delta_i$

$q = \mu+\tau\varepsilon_i(n)$

$i = 1,\ldots,I$

$\delta_i \sim {}_\tau\text{Student } t$

y_i = losses from the i^{th} cell

n_i = inventory in the i^{th} cell.

Then the maximum likelihood function is

$$L(\mu,\tau) = \prod\int\{e^{\mu+\tau x}/[(1+e^{\mu+\tau x})(1+(x/\tau)^2/n)^{(n+1)/2}]\}dx \qquad \text{(F.2)}$$

where $i = 1,\ldots,n$, and the integration is performed over the real line. This likelihood function is used to find μ and τ that maximize $L(\mu,\tau)$.

Next, if we assume y_i, and let $y = y_i$, then the expected value of p_i is

$$E[p_i] = \int p_i(x)p_i{}^y(1-p_i)^r \, c(n)/[1+(x/\tau)^2/n]^{(n+1)/2}dx, \qquad \text{(F.3)}$$

where $r = n_i-y_i$, $c(n)$ is the appropriate normalizing constant, and the integration is performed over the real line. This expected value is the empirical Bayes estimator of p_i.

See Deely and Lindley [Ref. 9] for more on empirical Bayes.

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10.	Commandant of the Marine Corps HQMC, Code MPI-40 Washington, D.C. 22314	1
11.	Commanding Officer Navy Personnel Research and Development Center San Diego, California 92152	1
12.	Major John R. Robinson Center for Naval Analysis MCOAG 4401 Ford Avenue Alexandria, Virginia 22302-0268	5

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c.1 Limited translation shrinkage estimation of loss rates in Marine Corps manpower models.